Essentializing Equilibrium Concepts

Julio González-Díaz

Research Group in Economic Analysis
Universidad de Vigo
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(joint with Federica Briata, Ignacio García-Jurado and Fioravante Patrone)

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Initial Motivation

The example that lead to this research

Repeated games

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- Take the finitely repeated prisoner's dilemma

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Adding commitments greatly increases the size of the game tree

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Outline

1. Motivation
2. Definitions and Results
   - Preliminary notations
   - Main definitions
   - Results
3. An Example

Outline

Motivation

What do we mean by essentialize?

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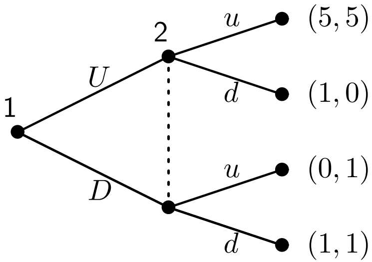

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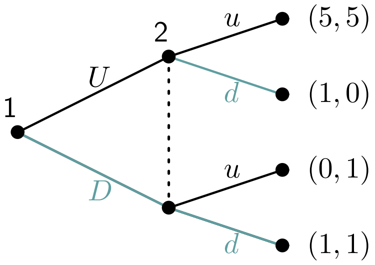

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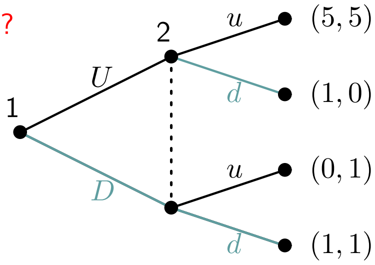

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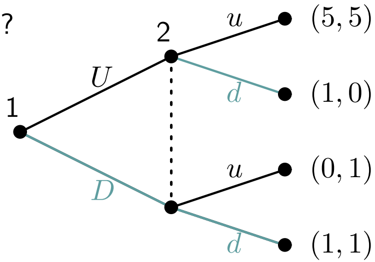

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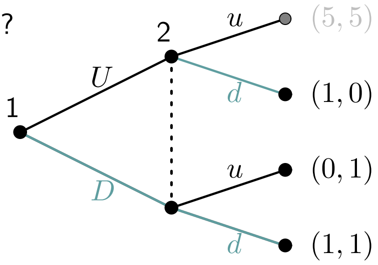

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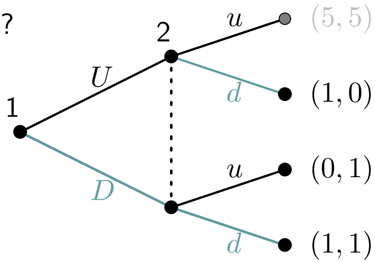

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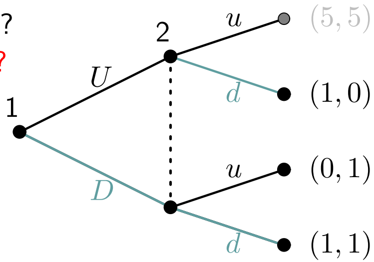

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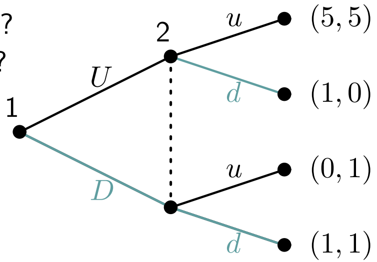

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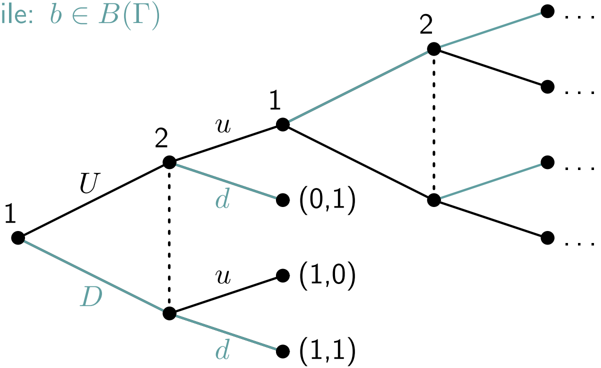

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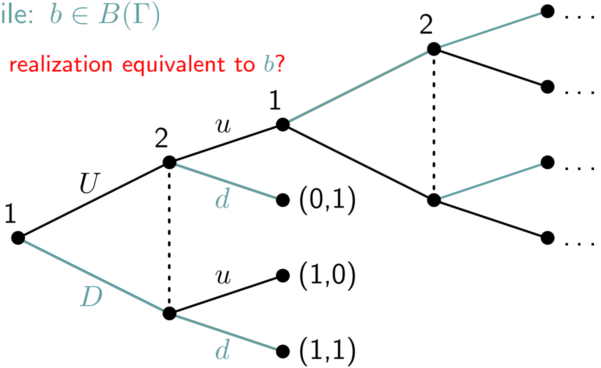

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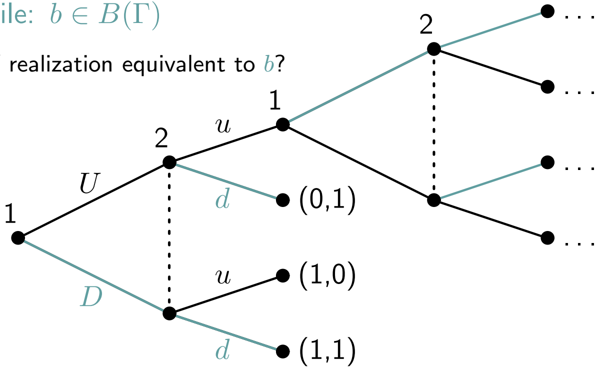

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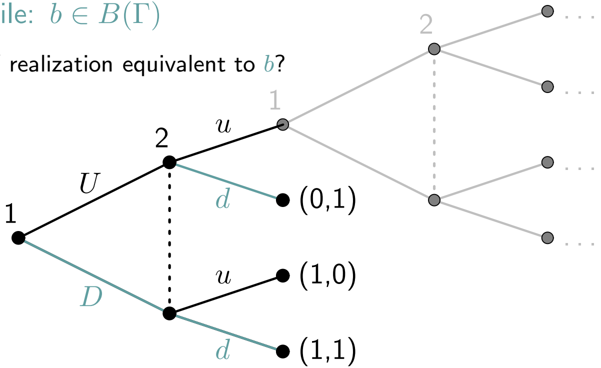

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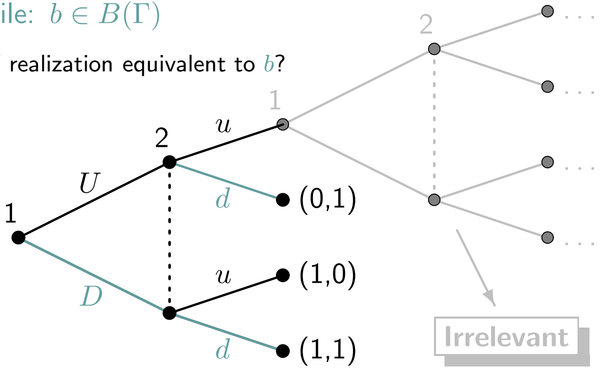

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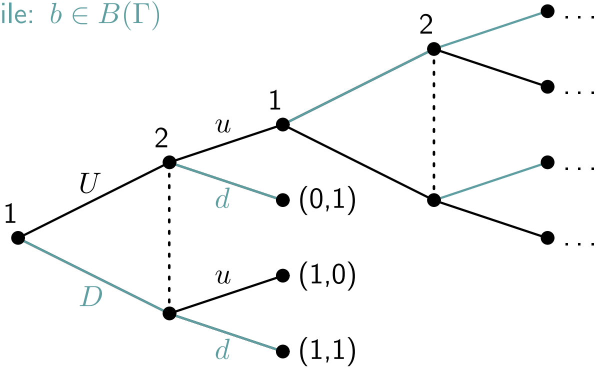

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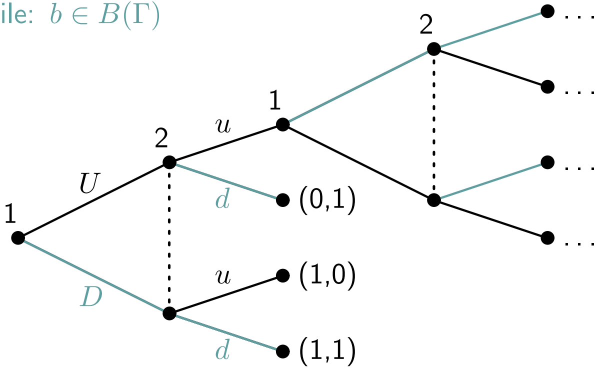

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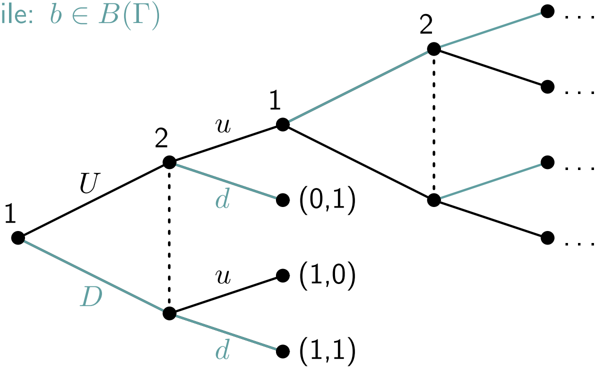

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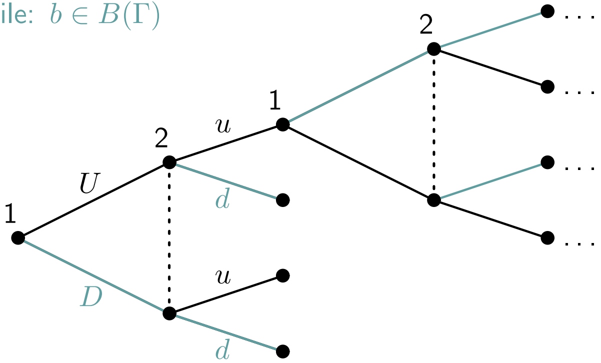

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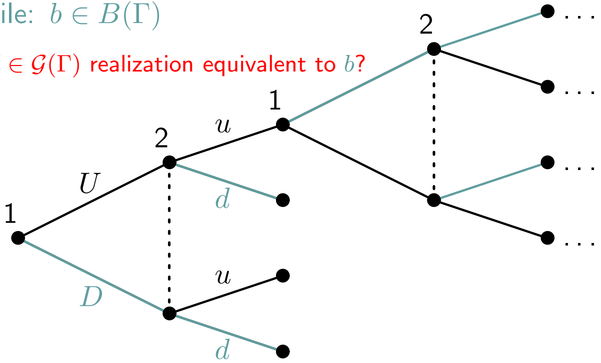

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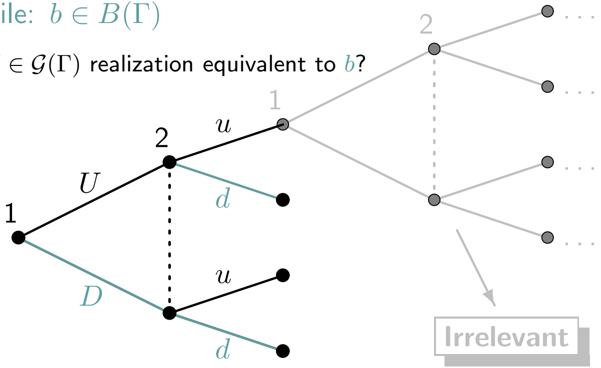

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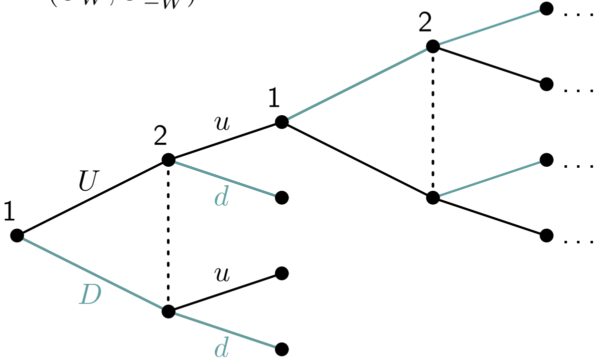

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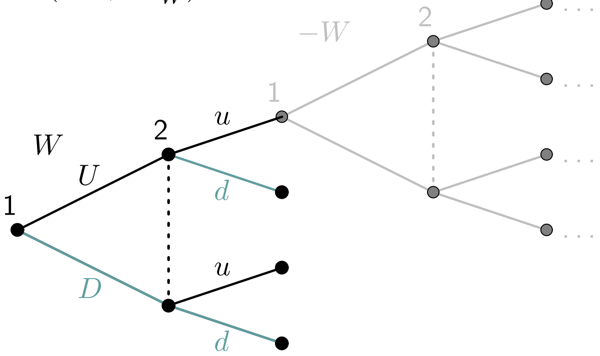

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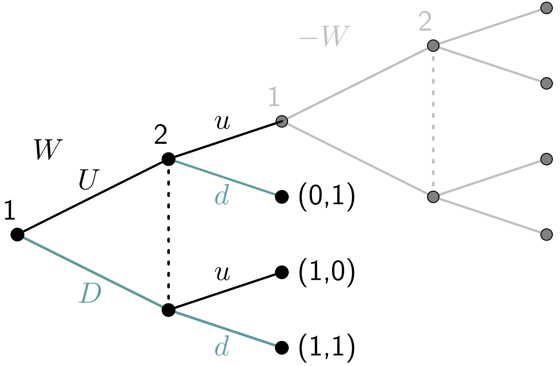

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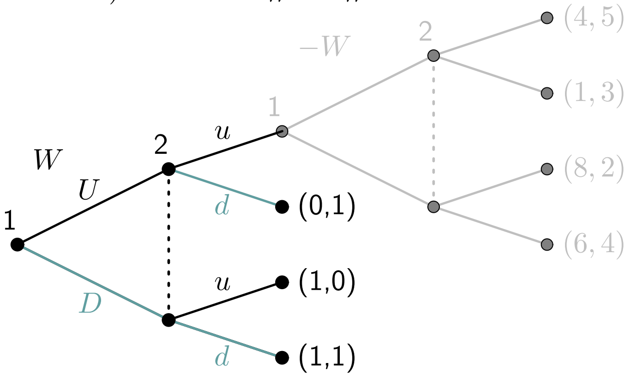

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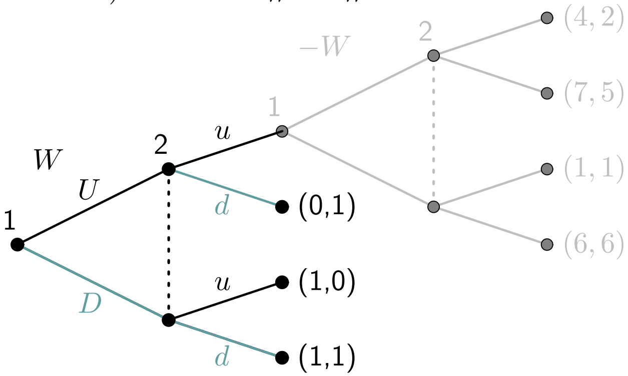

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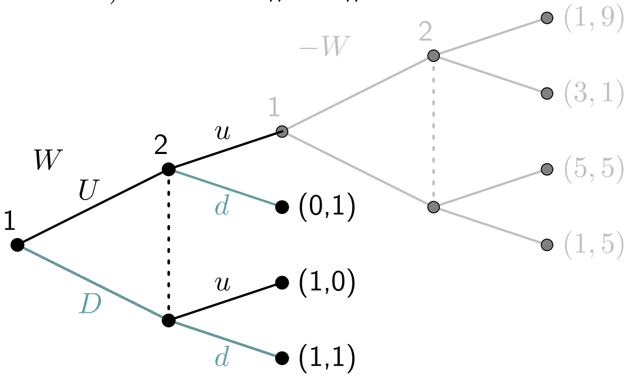

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- To essentialize an equilibrium concept EC is to find a map that assigns, to each pair (Γ, b), the essential collection $W_{\mathrm{EC}}(\Gamma, b)$

Reduced Game

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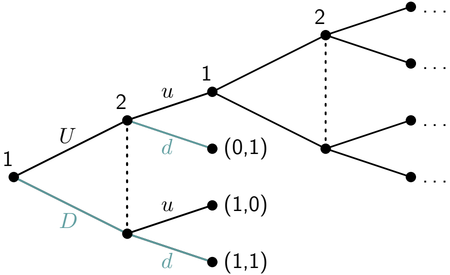

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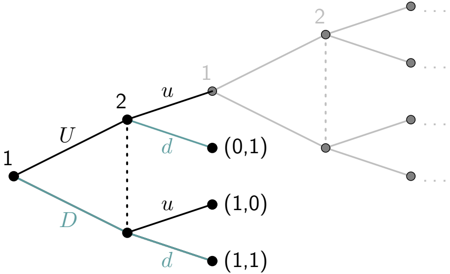

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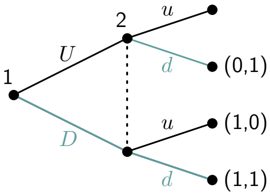

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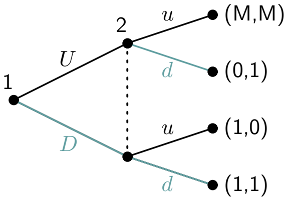

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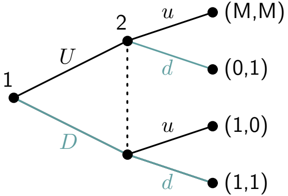

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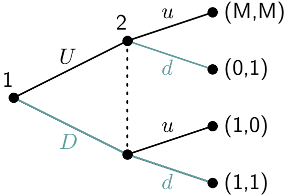

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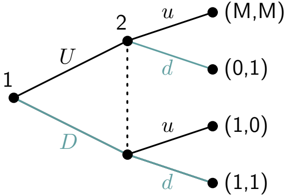

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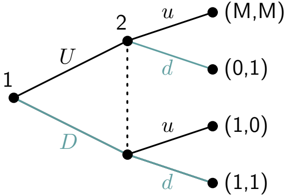

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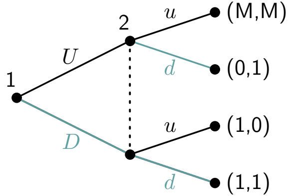

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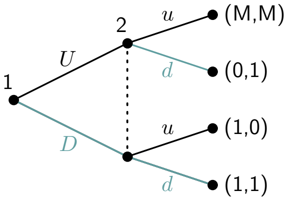

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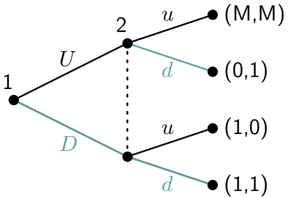

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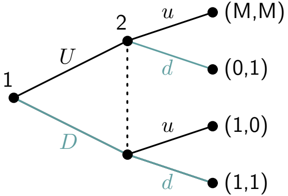

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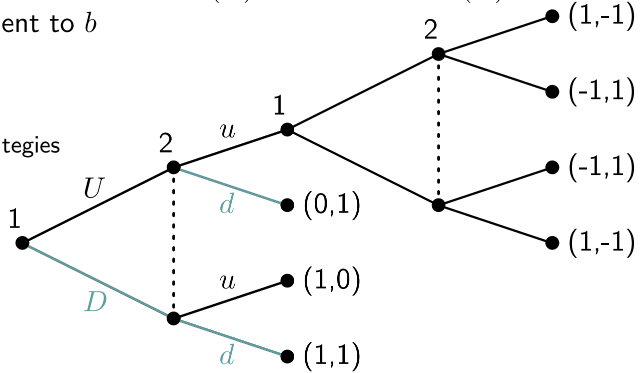

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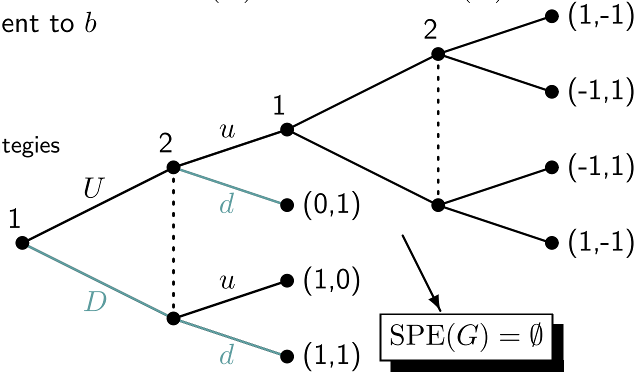

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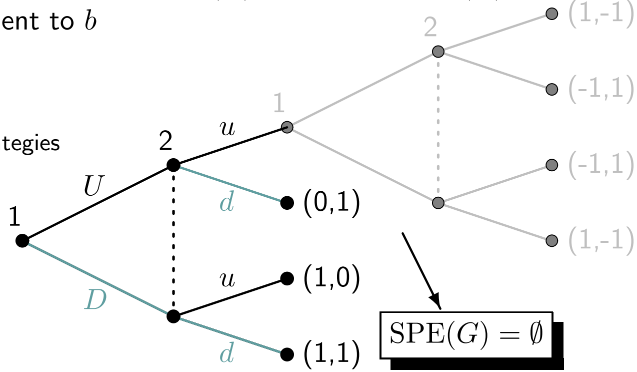

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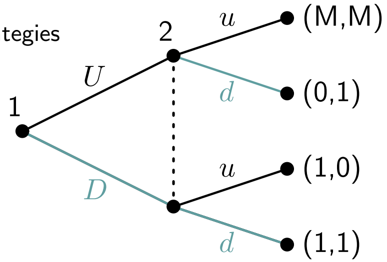

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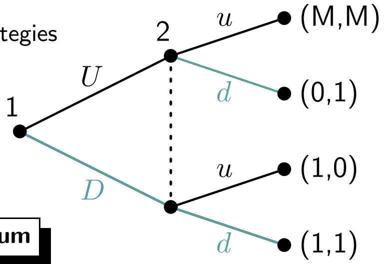

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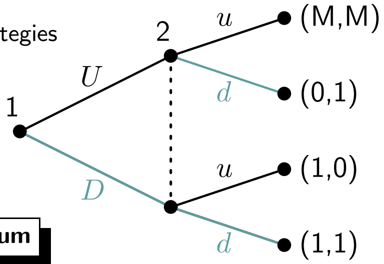

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Closedness requirement??

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The more demanding the EC, the larger the essential collections

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—An assessment (b, μ) is sequentially rational if, given μ, all players are best replying at all information sets

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Possible versions of perfect Bayesian?

Essentializing (belief based) equilibrium concepts

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Our approach applies to equilibrium concepts that are sequentially rational under some f

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The more demanding the EC, the larger the essential collections

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Inclusions of essential collections

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$$W_{\mathrm{NE}} \subset W_{\mathrm{SPE}}$$

Inclusions of essential collections

$$W_{\mathrm{NE}} \subset W_{\mathrm{SPE}} \qquad W_{\mathrm{SE}} \subset W_{\mathrm{WPBE}}$$

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Inclusions of essential collections

$$W_{\mathrm{NE}} \subset W_{\mathrm{SPE}} \subset W_{\mathrm{SE}} \subset W_{\mathrm{WPBE}} \subset W_{\mathrm{SR}} = W_{\mathrm{PE}} = U$$

Outline

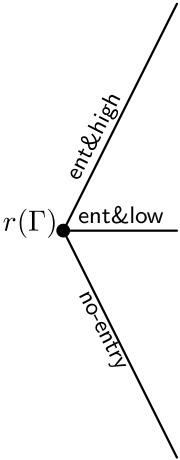
1. Motivation
2. Definitions and Results
   - Preliminary notations
   - Main definitions
   - Results
3. An Example

Licensing Auction

Players

1. Foreign Firm
2. Government Official
3. Local Firm

Licensing Auction



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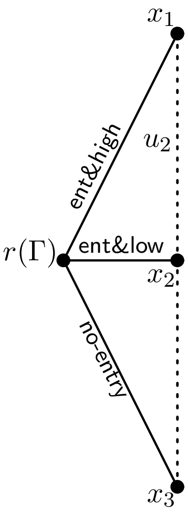

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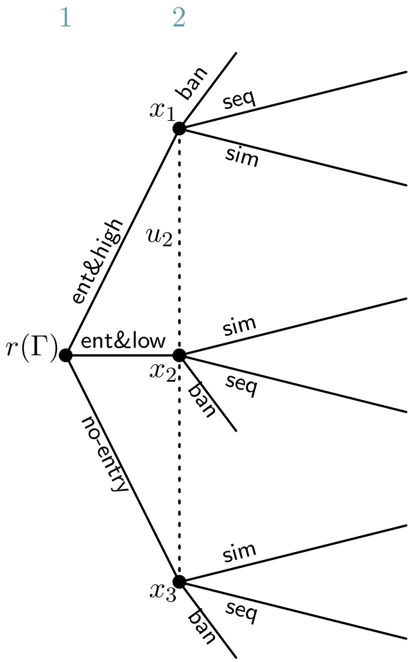

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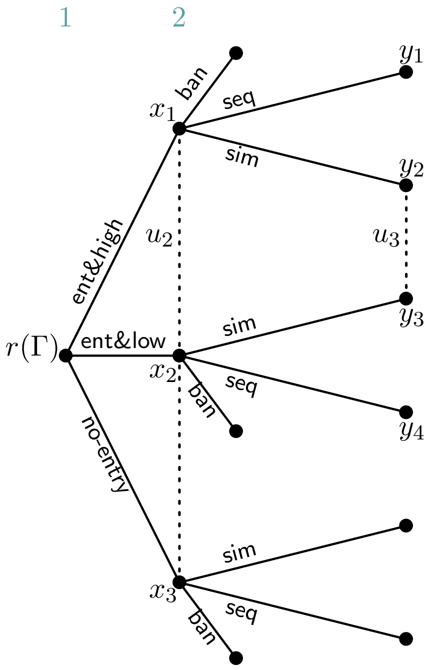

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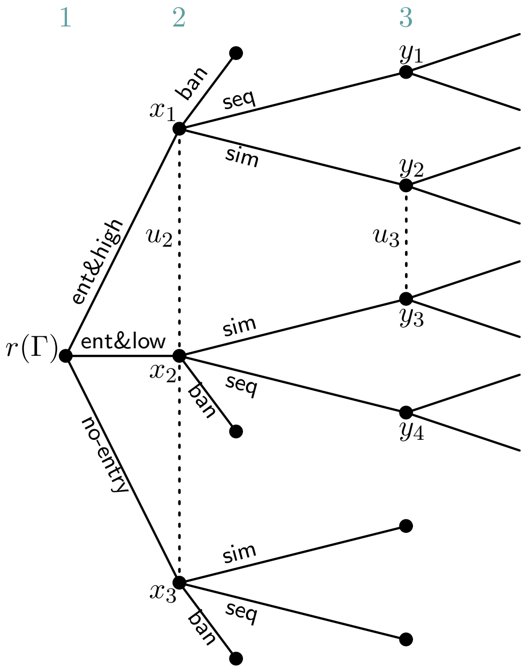

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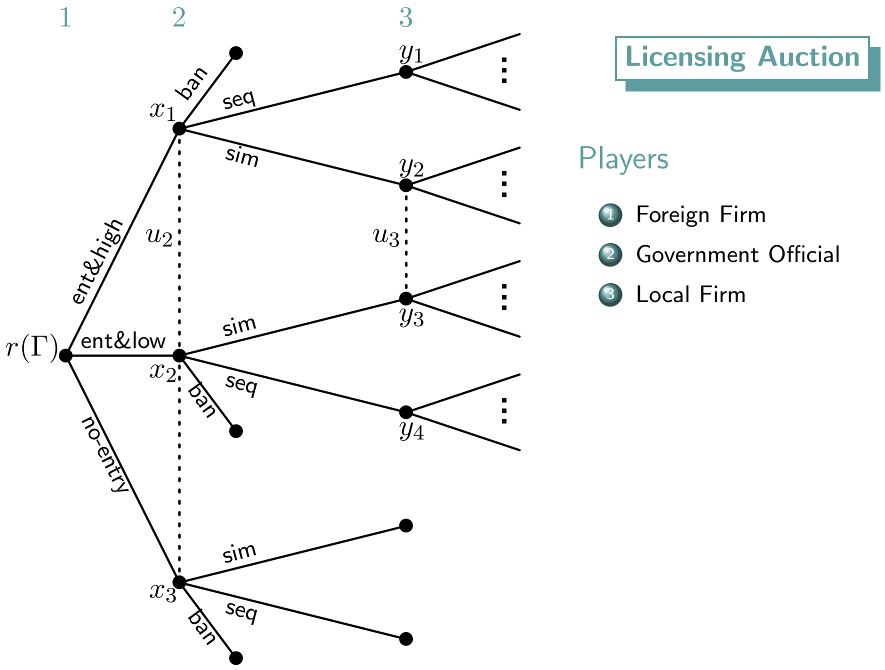
1
2
3
Licensing Auction
y_1
y_2
y_3
y_4
x_1
x_2
x_3
u_2
u_3
$r(\Gamma)$
ban
seq
sim
ent&high
ent&low
no-entry
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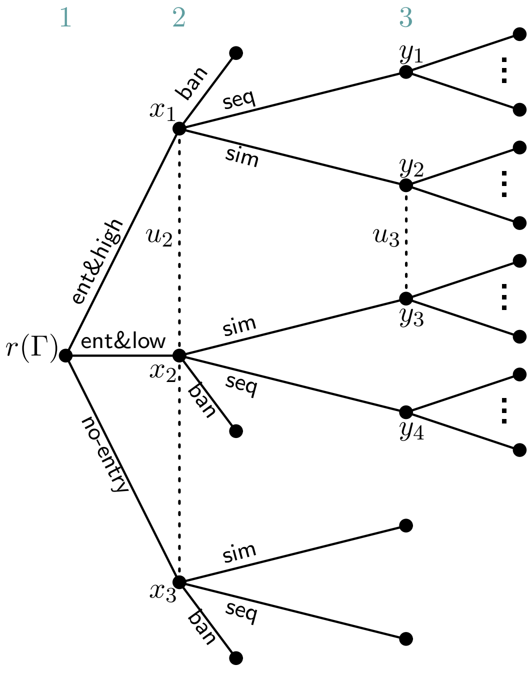

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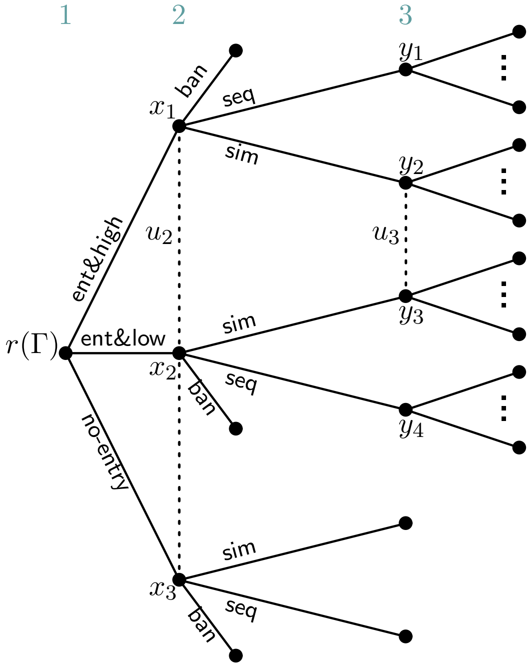

Players

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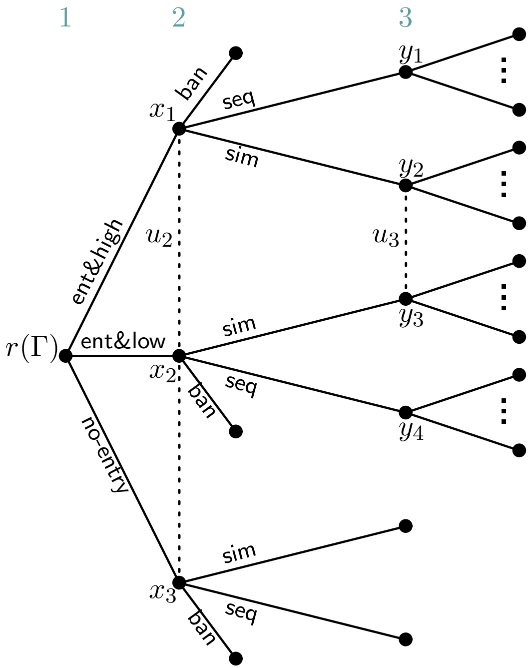
Features



Licensing Auction

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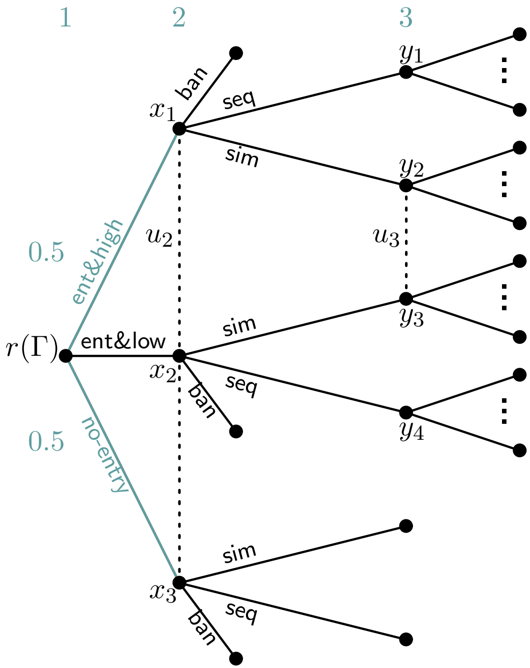
Features

- Strategies



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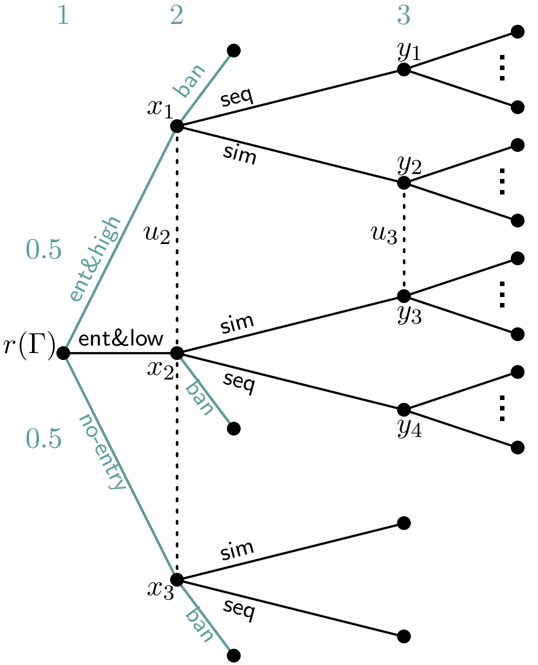

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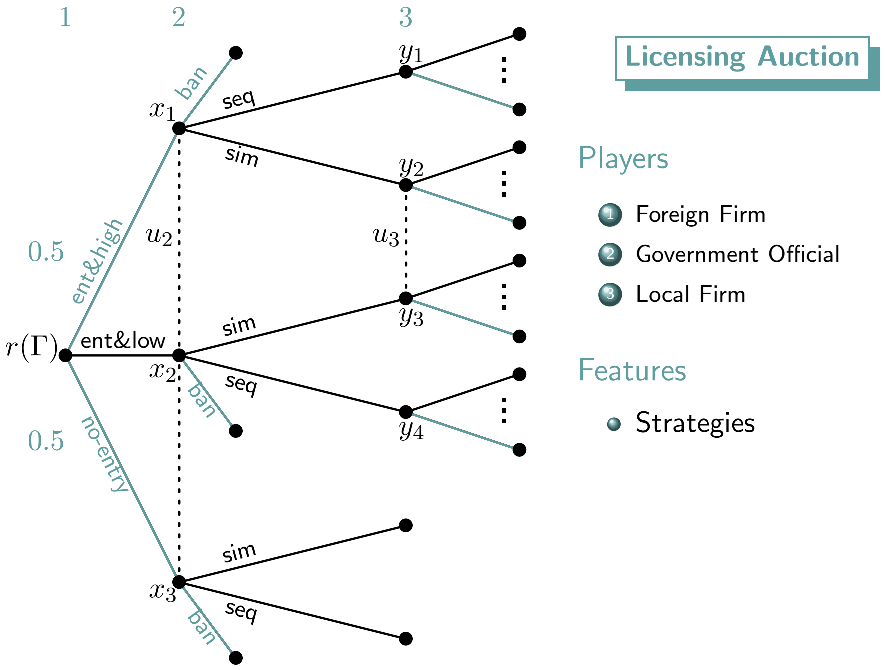
Licensing Auction
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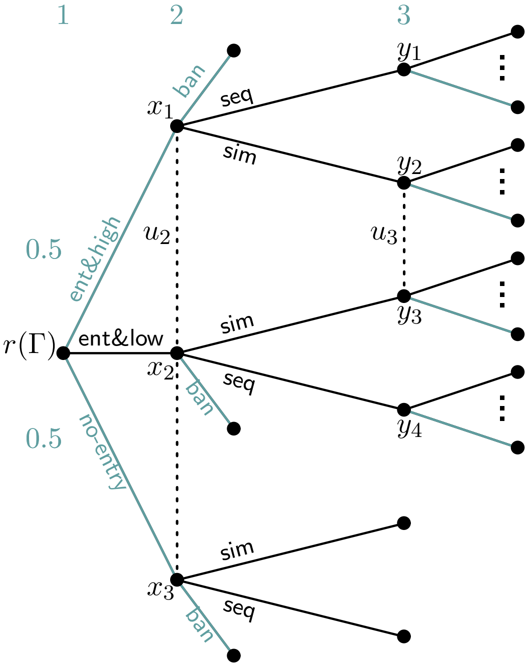

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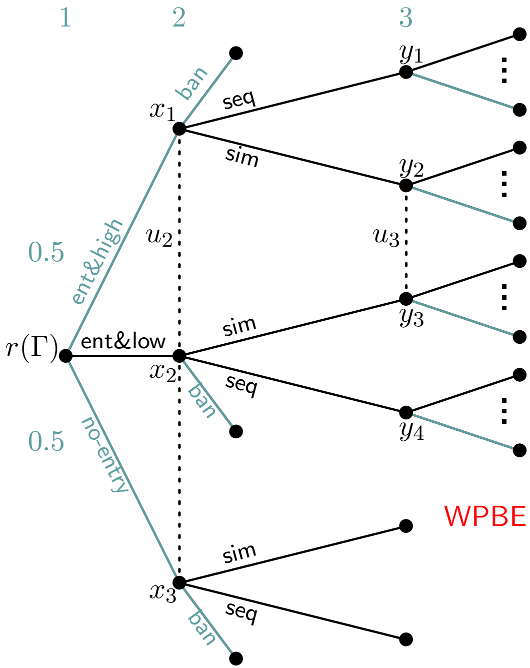
- Essential collections



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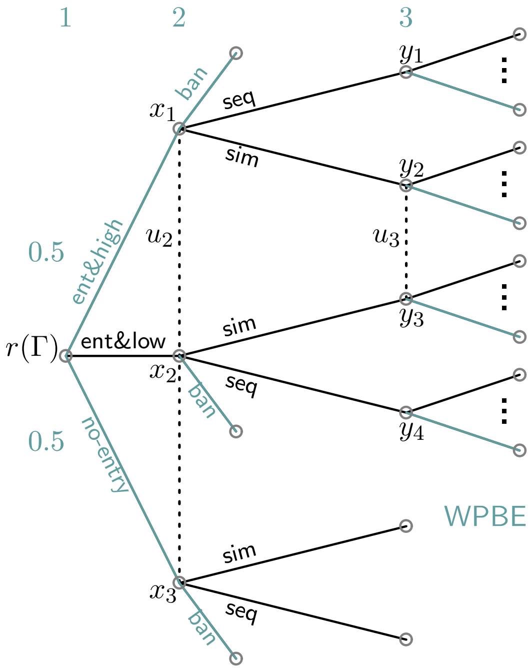

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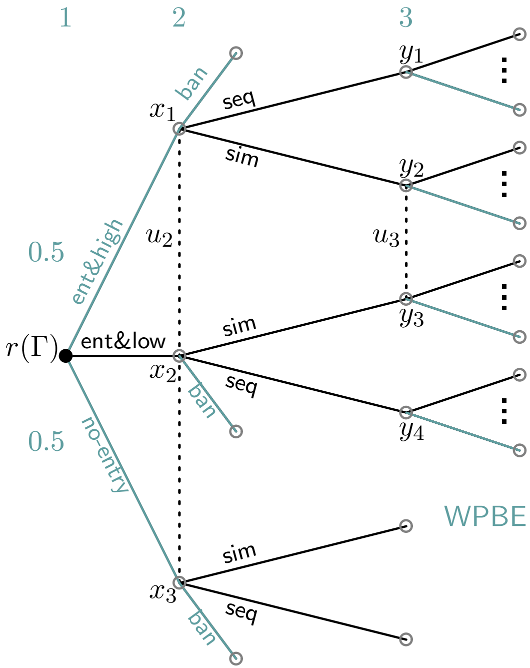

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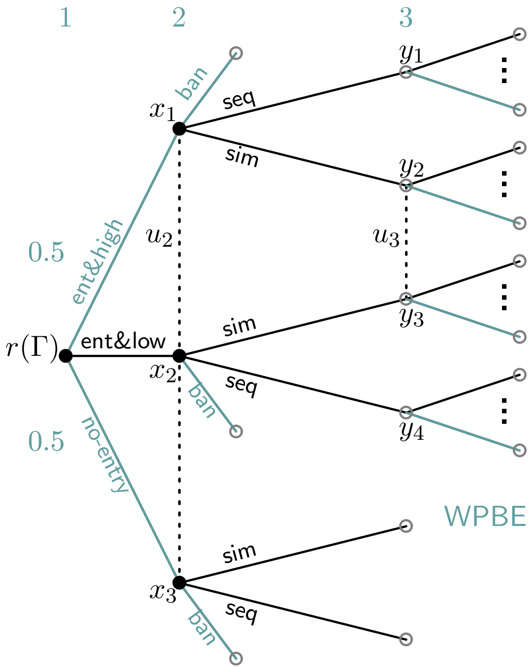

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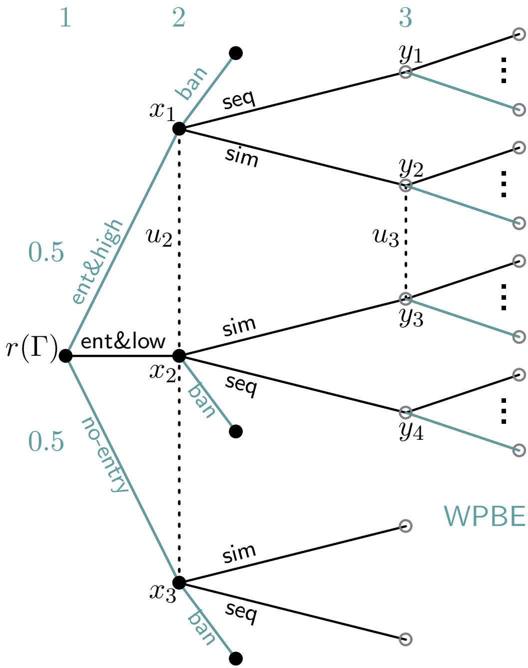

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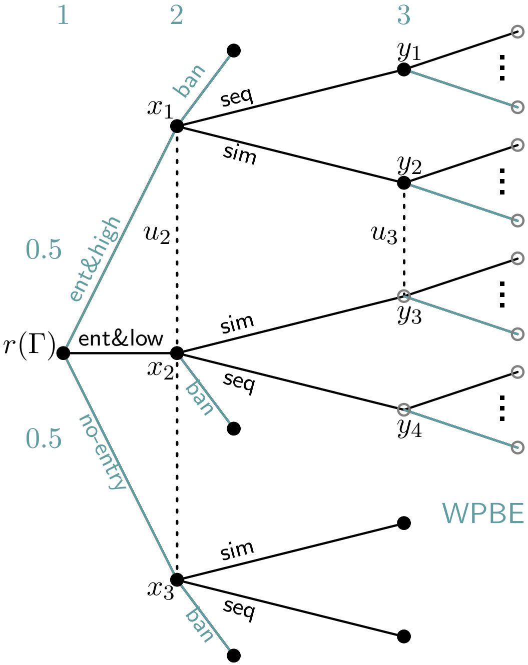

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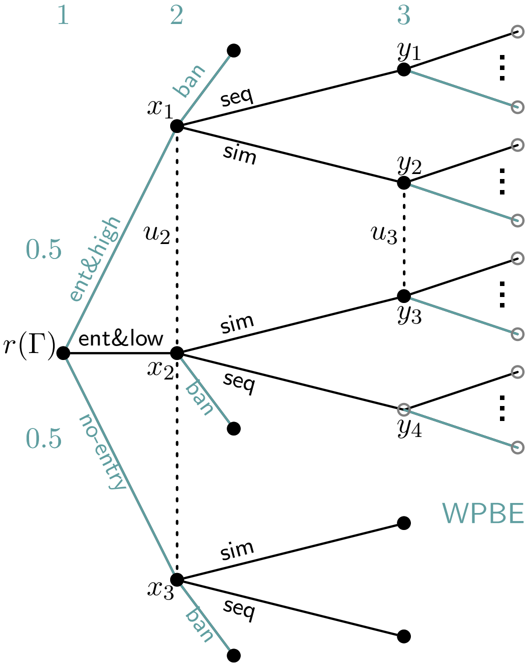

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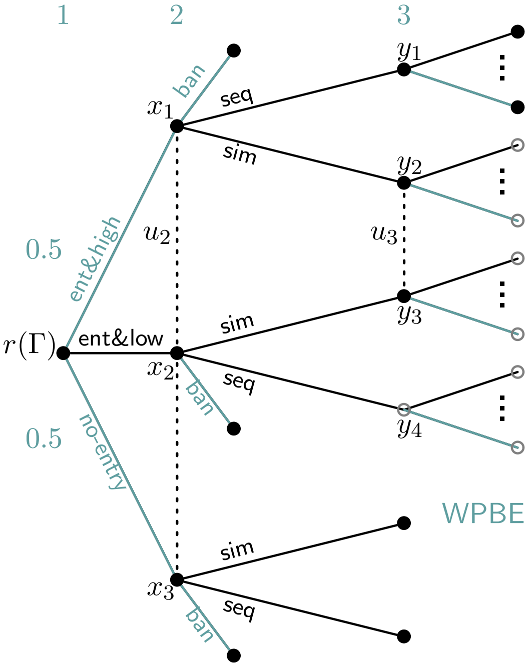

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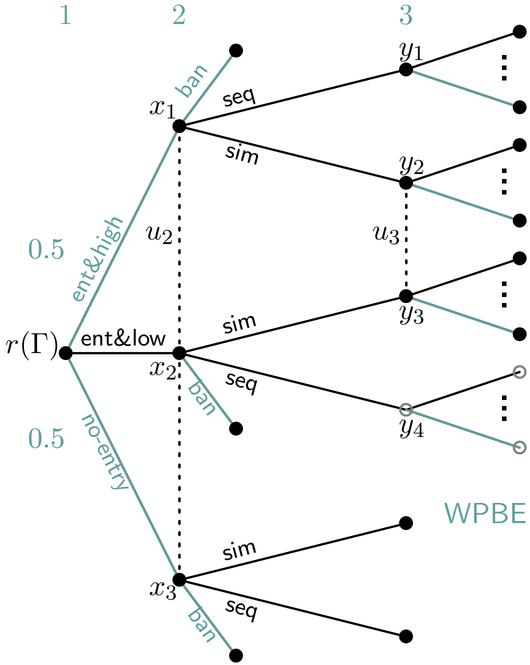

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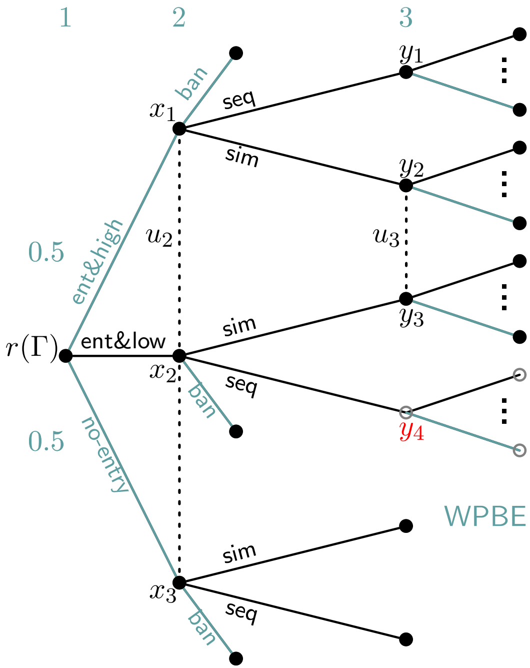

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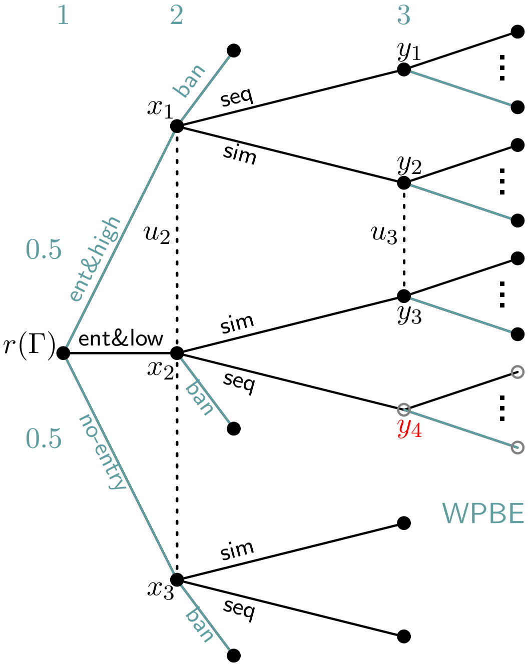
Licensing Auction



WPBE

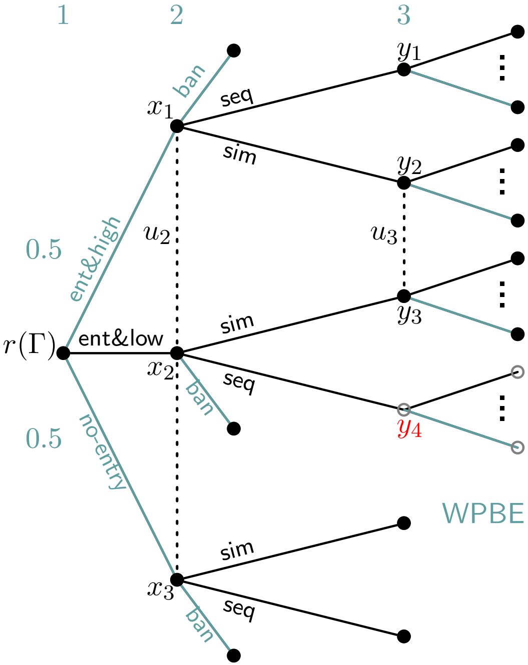
Players

1. Foreign Firm
2. Government Official
3. Local Firm

Features

- Strategies
- Essential collections
- Reduced game

Licensing Auction

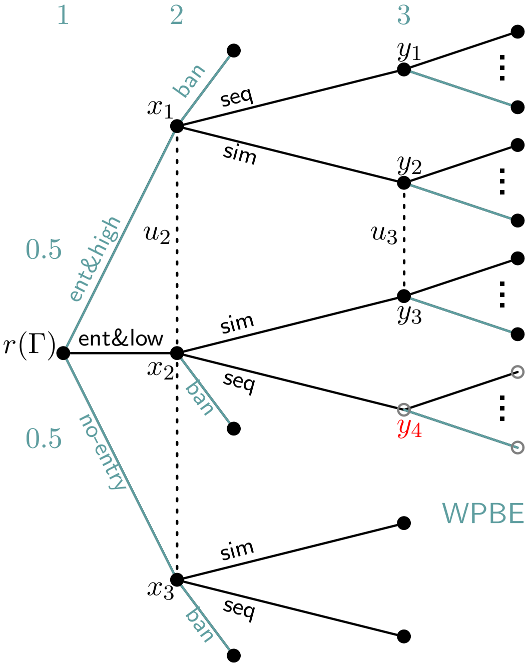


Players

1. Foreign Firm
2. Government Official
3. Local Firm

Features

- Strategies
- Essential collections
- Reduced game
- Structural robustness



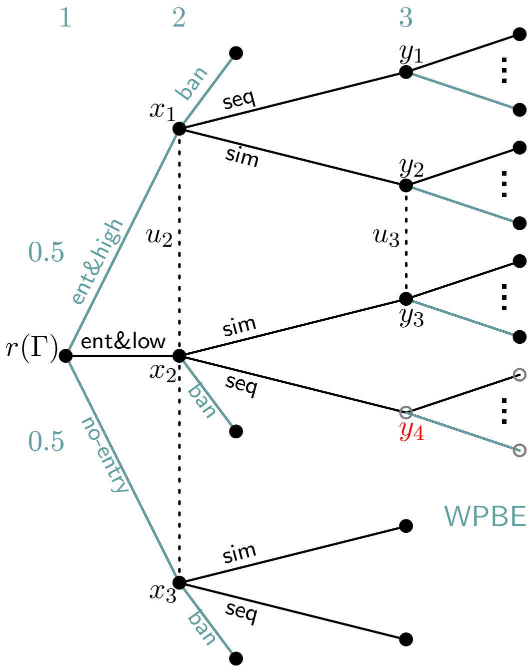
Licensing Auction

Players

1. Foreign Firm
2. Government Official
3. Local Firm

Features

- Strategies
- Essential collections
- Reduced game
- Structural robustness
- Partial specifications



Licensing Auction

Players

1. Foreign Firm
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Features

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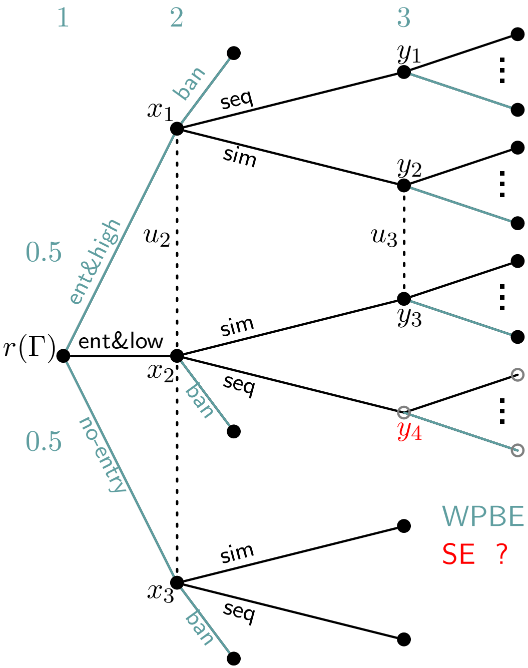
- Essential collections
- Reduced game
- Structural robustness
- Partial specifications
- Virtual equilibria

Licensing Auction



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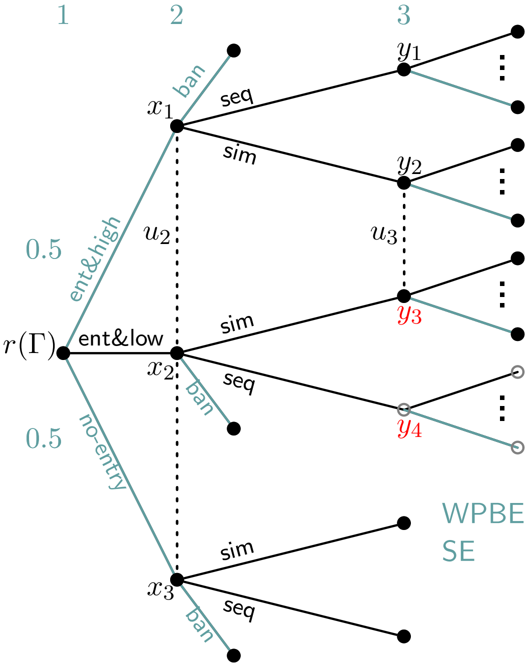

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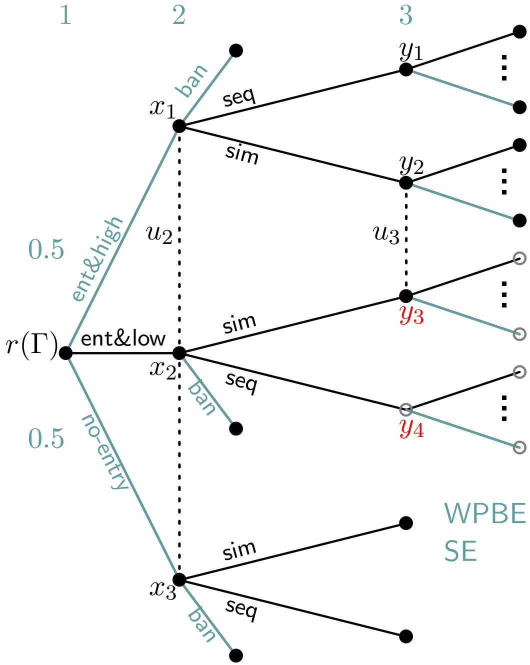

WPBE
SE

Licensing Auction

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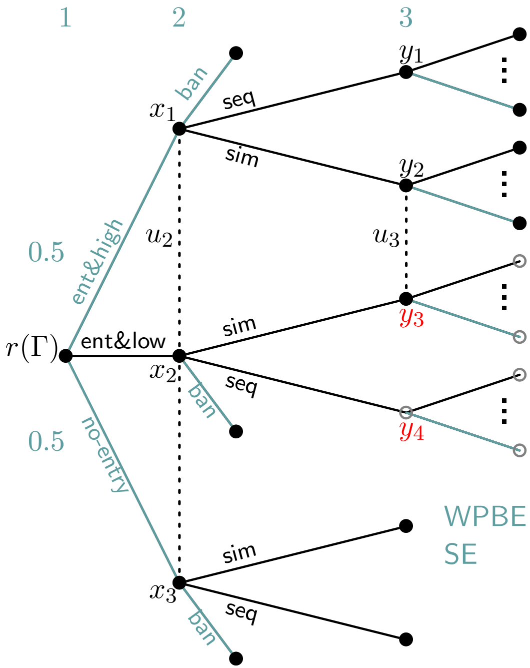

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Essentializing Equilibrium Concepts

Julio González-Díaz

Research Group in Economic Analysis
Universidad de Vigo
....

(joint with Federica Briata, Ignacio García-Jurado and Fioravante Patrone)

February 2th, 2009