Gas transmission networks in Europe: Connections between different entry-exit tariff methodologies ☆

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Abstract

Following a request of the European Commission in 2012, different bodies within the gas energy sector have been working on a Network Code for transmission tariffs. The final goal is to get a more harmonized structure within the European Union. This paper complements those efforts by developing a formal treatment of some methodological aspects arising in past and present drafts of the Network Code.

First, the analysis provides simple formulas for the computation of the tariffs resulting from the application of two of the main methodologies that have been discussed in the official documents: the capacity-weighted distance approach and the least squares approach. Second, it is shown that the tariffs delivered by the two approaches are perfectly correlated with each other. Maybe more importantly, if a natural adjustment is performed to control tariff dispersion, then both approaches lead to exactly the same tariffs.

Moreover, the analysis highlights an issue that may have been overlooked by regulators and also by past publications: the difference between weighted and unweighted versions of the methodologies under study and the reasons why weighted versions should be preferred. The paper concludes with a brief comparison with other methodologies and discussing some policy implications.

Keywords: gas networks, entry-exit tariffs, least squares, capacity-weighted distance, weighted methodologies, European regulations

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1. Introduction

Since the 3rd EU Energy Package entered into force in 2009, there has been a growing interest in the design of the access tariffs to the different transmission networks in the European Union. This interest has led to an increase in the related literature, which ranges from reports and regulations at the national and European levels to more academic papers published in peer-reviewed journals. In these contributions the emphasis is normally put on the so called *entry-exit methodologies*, which assign tariffs to all entry and exit points of the network. Thus, the final tariff associated with a given flow depends both on the chosen point of entry and on the destination of the flow.

Regulation no. 715/2009 of the European Commission prescribes that the methodologies used to calculate tariffs should be transparent, cost-reflective, non-discriminatory and, moreover, should preserve system integrity and provide appropriate return on investment. Following a request of the European Commission in June 2012, and building upon the guidelines in the above regulation and related ones, the Agency for the Cooperation of Energy Regulators elaborated the *"Framework Guidelines on Rules Regarding Harmonised Transmission Tariff Structures in European Gas Transmission Networks"* (ACER 2013), hereafter FG-2013. Four main methodologies were proposed in this document. FG-2013 was then submitted to the European Network of Transmission System Operators for Gas (ENTSOG), who prepared several drafts of the Network Code since then. The last one of such documents was released in July 2015 (ENTSOG 2015) and sent to the European Commission, who published a new draft in February 16 (EC 2016), hereafter NC-2016. The process is now in its final stages and a regulation from the EU regulating tariff design in gas transmission networks should be approved soon.

The first of the methodologies discussed in FG-2013 is the traditional *postage stamp methodology*, but it is only considered acceptable under special circumstances (its main drawback is that it is not cost-reflective). A second methodology, named *virtual point-based approach*, is based on marginal costs and is very similar to the *long run marginal cost methodology* that has been in place for several years in the UK (see, for instance, the report of the National Grid (2011)). The other two methodologies, called *capacity-weighted distance approach* and *matrix approach*, are built upon average costs. Since the matrix approach has already been widely discussed in the literature under the name of *least squares approach*, the present paper also sticks to this name. For some papers on this methodology the reader may refer, for instance, to Deliberata (2006), Alonso et al. (2010), National Energy Comission of Spain (2012), Apolinário et al. (2012), and Bermúdez et al. (2013).

The focus of this paper is on the two methodologies based on average costs. One of the main contributions consists in formally developing the definitions in FG-2013, obtaining closed-form expressions to easily compute the associated tariffs. Importantly, relying on these expressions it can be shown that both the capacity-weighted distance and the least squares approaches deliver very similar tariffs. The previous claim is formalized in Section 5. In particular, it is shown that if a natural dispersion control is imposed on the tariffs as a secondary adjustment, then the two methodologies deliver exactly the same tariffs.

In the analysis, special attention is devoted to an aspect that is very relevant for an adequate tariff design and that has been overlooked by most of the literature so far.[1] Setting aside the postage stamp methodology, all other methodologies require to perform, in one way or another, computations that deal with averages associated to the entry and exit points in the network. These averages may be either weighted or unweighted, with the former taking into account that more important points should have more influence in the final average. This paper presents some arguments in favor of the weighted versions of the different methodologies. It is worth noting that the two methodologies of the original FG-2013 document in which this issue was not handled in a consistent way, virtual point-based and least squares approaches, are not present in NC-2016.

[1] An exception is Bermúdez et al. (2013).

Finally, Section 6 presents a brief comparison of the methodologies discussed in this paper with other tariff methodologies. Although this comparison is made on a simple example, it helps to get a sense for the differences between the approaches regarding potential policy implications.

To conclude this introduction it is worth mentioning that the main insights from this paper were presented to ENTSOG in early 2014.[2] Remarkably, the main suggestions that can be extracted from these insights have been incorporated into NC-2016, namely, i) removal of one of the two methodologies that have been shown to be essentially equivalent and ii) disregarding the unweighted versions of the discussed methodologies.

2. Related literature: contribution to the state of the art

Academic research on energy networks is rapidly growing. In particular, the increasing consumption of natural gas within the European Union has led to an even sharper growth of the literature on this specific source of energy. Research focuses on a wide variety of topics such as broad regulatory aspects (Percebois, 1999; Jamasb et al., 2008; Spanjer, 2008), security of supply and socio-economic risks (Doukas et al., 2010, 2011), optimization models accounting for operational costs of the transmission network (Martin et al., 2006; Ríos-Mercado and Borraz-Sánchez, 2015, and references therein) and network expansions (Dieckhöner et al., 2013; Chaudry et al., 2014; Üster and Dilaveroğlu, 2014; Zhang et al., 2015).

This paper deals with tariff design, which is another important aspect in transmission networks.[3] More specifically, it studies the so-called entry-exit tariffs in the specific context of gas networks. Related aspects have also been discussed for electricity networks, but normally from a very descriptive perspective, dealing with specific implementations and not so much with

[2] This presentation was made by one of the current authors in a meeting of ENTSOG's tariffs working group in Brussels.

[3] It is worth mentioning that tariff design has also been studied in distribution networks. Yet, given the "proximity" to the final consumer, regulations and general objectives are of a different nature. Thus, distribution networks are normally dealt with independently (refer to Bernard et al. (2002) and Ramírez and Rosellón (2002)).

normative approaches (Malik and Al-Zubeidi, 2006; Lusztig et al., 2006). The recent European regulations, promoting the use of entry-exit tariffs, have led to the appearance of more detailed models and methodological discussions within the context of gas transmission networks.[4]

This paper contributes to the literature on tariff design by studying the capacity-weighted distance and the least squares approaches. The latter has already been studied before (Alonso et al., 2010; Apolinário et al., 2012; Bermúdez et al., 2013), but we do not know of any formal analysis of the former one.

2.1. Contribution to the state of the art

To the best of our knowledge this is the first paper in which a formal comparison of the two above methodologies is developed and, more importantly, the first one noticing that the two methodologies yield very similar tariffs. Thus, the approach in NC-2016, in which only one of them is included, seems more natural than having both of them as in FG-2013.

From the computational point of view, there is also an important addition to the existing literature. In previous works and regulations, the calculation of the tariffs associated with the least squares methodology required to solve an optimization problem. This task may be computationally demanding and, moreover, the problem is known to have infinitely many optimal solutions. The closed-form expressions obtained in this paper allow to easily characterize the solution set. More importantly, with them one can effortlessly compute the unique optimal solution associated with each desired entry-exit split.[5]

On the other hand, this paper raises an issue that is quite relevant for its policy implications: the use of weighted or unweighted methodologies. A formal analysis is presented, along with arguments in favor of weighted

[4] Interestingly, some of these discussions do not deal with the properties of specific entry-exit schemes, but with the overall limitations of the entry-exit model; see, for instance, Hewicker and Kesting (2009) and Hallack and Vázquez (2013).

[5] The entry-exit split specifies how much of the total revenue has to be collected at entry points and how much at exit points (see Section 3.2).

methodologies. This contrasts with the common practice nowadays, since most of the papers and regulations, including FG-2013, rely mainly on unweighted methodologies (Deliberata, 2006; Alonso et al., 2010; National Energy Comission of Spain, 2012; Apolinário et al., 2012). Two exceptions are Bermúdez et al. (2013) and NC-2016.

Finally, the analysis also includes a comparison between the entry-exit methodologies discussed in this paper and other tariff methodologies that have been discussed in the literature. Although the comparison is performed on a simple network, it delivers some insights regarding the general behavior of the different approaches. It is worth noting that these kind of comparisons are rarely seen in the literature. This is mainly because the different methodologies have different requirements and computations sometimes are far from being straightforward. Fortunately, the formal framework developed here facilitates the analysis.

Overall, the main contribution of this paper is to improve the understanding of the entry-exit methodologies that are nowadays being discussed by the European regulators. This understanding is important, since the ensuing regulations will be the basis for the tariffs implemented in the different countries in the European Union and possibly also taken as references at a global level.

3. Background

One of the main goals of this paper is to discuss the connections between two different entry-exit methodologies: the *capacity-weighted distance approach* and the *least squares approach* (Deliberata, 2006; Alonso et al., 2010; Apolinário et al., 2012; Bermúdez et al., 2013). Although both of them were discussed in FG-2013, only the former appears in NC-2016.[6]

Figure 1 displays the technical framework in which the methodology analysis in this paper is developed. We abstract away from some aspects of

[6]In FG-2013 the least squares methodology is discussed under the name of *matrix approach*.

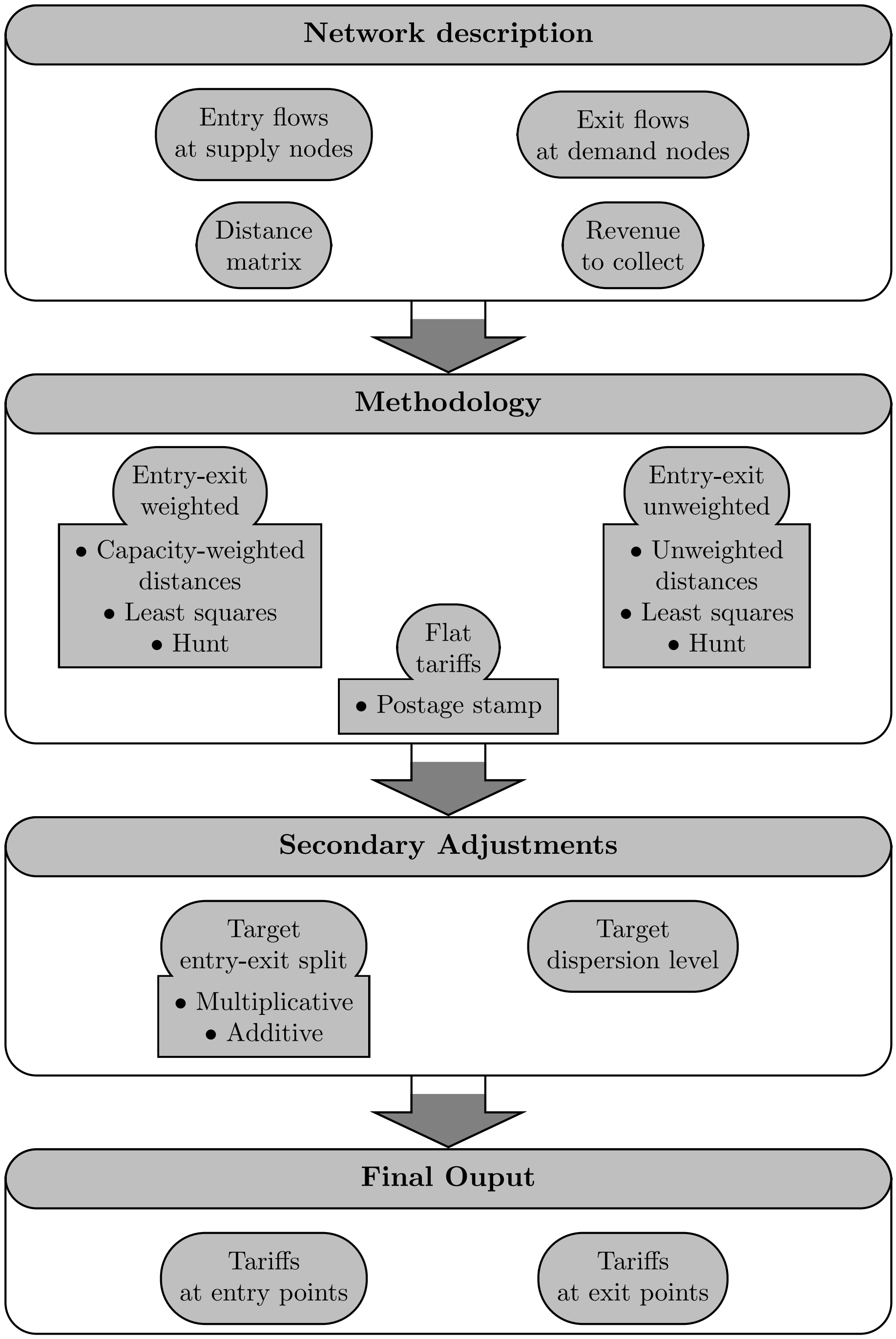

Figure 1: Technical framework for the analysis of tariff methodologies.

the entry-exit methodology that are not relevant for the analysis, such as the computation of the costs associated with the different infrastructures of the system or the revenue to be collected with the tariffs. Thus, it is assumed that there is a gas network with n entry points and m exit points and a reference scenario (average/high/peak demand) from which the following elements have already been identified:

- A *cost or distance matrix* $\mathcal{C}_{n\times m}$ such that each entry $\mathcal{C}_{ij}$ is a measure of the cost incurred when moving a unit of flow from entry point i to exit point j in the given scenario.[7]

- A strictly positive vector of *entry flows* $\boldsymbol{F}^{\mathbf{ET}} \in \mathbb{R}^n$ such that, for each entry point i, F_i^{ET} represents the flow entering the network at that point in the given scenario.

- A strictly positive vector of *exit flows* $\boldsymbol{F}^{\mathbf{XT}} \in \mathbb{R}^m$ such that, for each exit point j, F_j^{XT} represents the flow leaving the network at that point in the given scenario.

The goal is to determine a vector of entry tariffs, $\boldsymbol{ET} \in \mathbb{R}^n$, and a vector of exit tariffs, $\boldsymbol{XT} \in \mathbb{R}^m$. The joint application of these tariffs should collect the desired revenue R.

Assuming that the entry and exit flows are balanced, one can define $w = \sum_{i=1}^n F_i^{\mathrm{ET}} = \sum_{j=1}^m F_j^{\mathrm{XT}}$. Denote $\hat{C} = \sum_{i=1}^n \sum_{j=1}^m F_i^{\mathrm{ET}} F_j^{\mathrm{XT}} \mathcal{C}_{ij}$. Under the interpretation that each entry of matrix $\mathcal{C}$ represents the cost of sending one unit of flow between a given pair of entry and exit points in the system, $\sum_{i=1}^n F_i^{\mathrm{ET}} \mathcal{C}_{ij}$ would represent the cost of sending all flows to exit point j. Then, since the capacity of j is just F_j^{XT}, it is natural to think that the cost associated to j is

$$(\sum_{i=1}^n F_i^{\mathrm{ET}} \mathcal{C}_{ij}) \frac{F_j^{\mathrm{XT}}}{\sum_{i=1}^n F_i^{\mathrm{ET}}}, \textit{i.e.},$$

[7]For a deeper exposition on the computation of this matrix, the reader may refer to any of the papers mentioned at the beginning of this section. For instance, the suggestion in NC-16 is to use the shortest pipeline distance between i and j (although it also allows to omit those pairs of points which are unlikely to send flow to one another).

node j is only "responsible" for the proportion of the total flows that use this exit point. Adding up the above expression for all j we get

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m} F_i^{\mathrm{ET}} F_j^{\mathrm{XT}} \mathcal{C}_{ij}}{\sum_{i=1}^{n} F_i^{\mathrm{ET}}} = \frac{\hat{C}}{w}.$$

Therefore, one can assume that the revenue to be collected in the system, R, is given by precisely the above amount:

$$R = \frac{\hat{C}}{w}.$$

This assumption can be made without loss of generality: given an alternative target revenue $\hat{R}$, the corresponding tariffs can be obtained multiplying by $\hat{R}/R$ the ones collecting R. It is worth emphasizing that all the derivations and results presented below are unaffected by this kind of rescaling. More precisely, any rescaling that consists in multiplying all the entry and exit tariffs by the same constant can be done at any stage of the computation.

3.1. Weighted and unweighted averages

Setting aside the postage stamp methodology, all other methodologies require to perform computations that deal with averages associated to the entry and exit points in the network. In these computations one can choose between weighted or unweighted averages. Consider, for instance, the averages of the rows and columns of matrix $\mathcal{C}$, which are specially important in the ensuing analysis. The unweighted averages are defined as follows:

$$\begin{aligned} \text{for each } i \in \{1,\dots,n\}, \quad \bar{U}_i^{\mathrm{ET}} &= \frac{\sum_{j=1}^{m} \mathcal{C}_{ij}}{m} \quad \text{and} \\ \text{for each } j \in \{1,\dots,m\}, \quad \bar{U}_j^{\mathrm{XT}} &= \frac{\sum_{i=1}^{n} \mathcal{C}_{ij}}{n}. \end{aligned}$$

To get the weighted averages one needs to define

$$\begin{aligned}
\text{for each } i \in \{1, \ldots, n\}, \quad \bar{C}_i^{\mathrm{ET}} &= \frac{\sum_{j=1}^{m} F_j^{\mathrm{XT}} \mathcal{C}_{ij}}{\sum_{j=1}^{m} F_j^{\mathrm{XT}}} \quad \text{and} \\
\text{for each } j \in \{1, \ldots, m\}, \quad \bar{C}_j^{\mathrm{XT}} &= \frac{\sum_{i=1}^{n} F_i^{\mathrm{ET}} \mathcal{C}_{ij}}{\sum_{i=1}^{n} F_i^{\mathrm{ET}}}.
\end{aligned}$$

The motivation behind weighted averages is quite natural: points whose associated flows are larger should have a larger influence on the final average. To illustrate, think of the computation of the center of a gas network like the one in Figure 2, with one entry point and nine exit points. One option is to compute a simple average of the coordinates of all the points in the network, resulting in a point very close to the exit points. On the other hand, one can decide to weight the coordinates of each point by the amount of flow that it usually demands/supplies. Now, the center would be around the mid point of the big pipe in the figure (provided that the flows in the system are balanced). This last point seems definitely more realistic, since it corresponds with the *center of mass* of the gas flowing through the system.

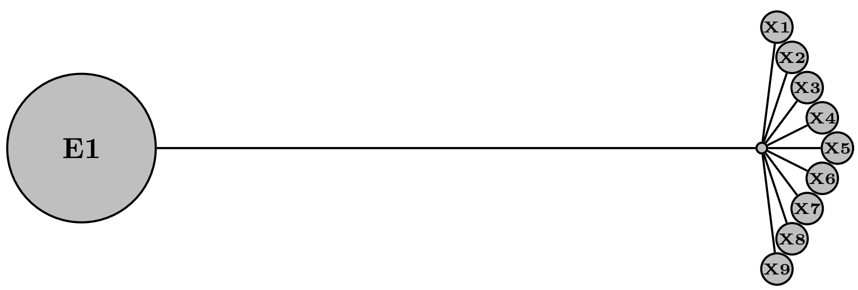

Figure 2: A gas network with one entry point and several exit points.

Section 6 shows that the same kind of effects illustrated above for the physical center of the network can arise when computing tariffs. In particular, unweighted methodologies lead to tariffs that are much more sensitive to network representations. Interestingly, the two methodologies of the original FG-2013 document in which this issue was not handled in a consistent way, virtual point-based and least squares approaches, are not present in NC-2016.

3.2. Tariff adjustment to obtain a target entry-exit split

An important aspect of an entry-exit methodology is the so-called *entry-exit split*, controlled by the split parameter s. Suppose that one has already computed vectors $\boldsymbol{ET}$ and $\boldsymbol{XT}$ of entry and exit tariffs, respectively. Yet, the final tariffs are required to collect s times more revenue at the entry points than at the exit ones. For instance, $s = 1$ corresponds with 50-50 split and $s = 3$ with 75-25 split, with three times more revenue collected via entry points than via exit points. There are two main options to adjust the $\boldsymbol{ET}$ and $\boldsymbol{XT}$ tariffs to accomplish the desired split: multiplicatively and additively.

3.2.1. Multiplicative adjustment

There are several equivalent ways of formulating this adjustment, maybe the most straightforward one is as follows: the multiplicative adjustment consists of finding $d \in [0,1]$ such that

$$\frac{\sum_{i=1}^{n} F_i^{\mathrm{ET}}(ET_i\, d)}{\sum_{j=1}^{m} F_j^{\mathrm{XT}}(XT_j(1-d))} = s.$$

Suppose that the initial tariffs delivered a 50-50 split, *i.e.*, $\sum_{i=1}^{n} F_i^{\mathrm{ET}} ET_i = \sum_{j=1}^{m} F_j^{\mathrm{XT}} XT_j$. Then, solving the above equation for d we get $d = \frac{s}{s+1}$ (and $1 - d = \frac{1}{s+1}$). Thus, the resulting tariffs are, for each entry point i and each exit point j,

$$ET_i^m = \frac{2s}{s+1} ET_i \quad \text{and} \quad XT_j^m = \frac{2}{s+1} XT_j. \tag{1}$$

All tariffs have been multiplied by two to ensure that the collected revenue remains unchanged.

3.2.2. Additive adjustment

The additive adjustment consists of finding $d \in \mathbb{R}$ such that

$$\frac{\sum_{i=1}^{n} F_i^{\mathrm{ET}}(ET_i + d)}{\sum_{j=1}^{m} F_j^{\mathrm{XT}}(XT_j - d)} = s.$$

Suppose that the initial tariffs delivered a 50-50 split. Then, solving this equation for d we get $d = \frac{s-1}{s+1}\frac{\hat{C}}{2w^2}$. Thus, the resulting tariffs are, for each entry point i and each exit point j,

$$ET_i^a = ET_i + \frac{s-1}{s+1}\frac{\hat{C}}{2w^2} \quad \text{and} \quad XT_j^a = XT_j - \frac{s-1}{s+1}\frac{\hat{C}}{2w^2}. \tag{2}$$

4. Tariff Methodologies

This section is devoted to present the formal definitions of the weighted versions of the *capacity-weighted distance approach* and the *least squares approach*. The latter, however, is normally seen in an unweighted form, even in the FG-2013 document (for an exception refer to Bermúdez et al. (2013)). Next, the definitions of the unweighted versions are briefly discussed. For the sake of completeness, the *postage stamp methodology* and a variation of the above methodologies that endogenously determines the entry-exit split are also defined. The postage stamp methodology is needed for the discussion in Section 5.1.

4.1. Capacity-weighted distance approach

This methodology uses vectors $\bar{\boldsymbol{C}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{C}}^{\mathbf{XT}}$ to define the entry and exit tariffs. The idea is quite natural: entry tariffs are proportional to the (capacity-weighted) average distance to the exit points. Similarly, exit tariffs are proportional to the corresponding average distance to the entry points. Only a minor modification is needed to ensure that the desired revenue is collected:

$$\boldsymbol{ET}^{\mathbf{C}} = \frac{\bar{\boldsymbol{C}}^{\mathbf{ET}}}{2} \quad \text{and} \quad \boldsymbol{XT}^{\mathbf{C}} = \frac{\bar{\boldsymbol{C}}^{\mathbf{XT}}}{2}.$$

Thus, the total collected revenue would be

$$\begin{aligned}\sum_{i=1}^{n} F_i^{\mathrm{ET}} ET_i^{\mathrm{C}} + \sum_{j=1}^{m} F_j^{\mathrm{XT}} XT_j^{\mathrm{C}} &= \sum_{i=1}^{n} F_i^{\mathrm{ET}} \frac{\sum_{j=1}^{m} F_j^{\mathrm{XT}} \mathcal{C}_{ij}}{2\sum_{j=1}^{m} F_j^{\mathrm{XT}}} + \sum_{j=1}^{m} F_j^{\mathrm{XT}} \frac{\sum_{i=1}^{n} F_i^{\mathrm{ET}} \mathcal{C}_{ij}}{2\sum_{i=1}^{n} F_i^{\mathrm{ET}}} \\ &= \frac{\hat{C}}{2w} + \frac{\hat{C}}{2w} \\ &= R.\end{aligned}$$

From this equation it can be seen that the above entry and exit tariffs naturally lead to a 50-50 split between entry and exit points. The multiplicative and additive adjustments for a given split parameter s are presented below.

4.1.1. Capacity-weighted distance approach with multiplicative adjustment

From Eq. (1) in Section 3.2.1 we get, for each entry point i and each exit point j,

$$ET_i^{\mathrm{C}_m} = \frac{2s}{s+1} ET_i^{\mathrm{C}} = \frac{s}{s+1} \bar{C}_i^{\mathrm{ET}} \quad \text{and} \quad XT_j^{\mathrm{C}_m} = \frac{2}{s+1} XT_j^{\mathrm{C}_m} = \frac{1}{s+1} \bar{C}_j^{\mathrm{XT}}. \tag{3}$$

4.1.2. Capacity-weighted distance approach with additive adjustment

From Eq. (2) in Section 3.2.2 we get, for each entry point i and each exit point j,

$$ET_i^{\mathrm{C}_a} = \frac{\bar{C}_i^{\mathrm{ET}}}{2} + \frac{s-1}{s+1} \frac{\hat{C}}{2w^2} \quad \text{and} \quad XT_j^{\mathrm{C}_a} = \frac{\bar{C}_j^{\mathrm{XT}}}{2} - \frac{s-1}{s+1} \frac{\hat{C}}{2w^2}. \tag{4}$$

4.2. Weighted least squares approach

Recall that one of the goals of tariff design is to have cost-reflective tariffs. Thus, one would like to have tariffs such that, when sending a unit of flow from point i to point j, the cost $\mathcal{C}_{ij}$ is collected. Equivalently, one would like to have that $ET_i + XT_j = \mathcal{C}_{ij}$ for each pair (i, j) of entry-exit points. However, in general this yields a system of equations with far more constraints than variables. The idea of the least squares approach is to choose tariffs minimizing the squared sum of the deviations with respect to the above equations. Following the proposal in Bermúdez et al. (2013), weights are included, so that deviations involving pairs of entry and exit points with large associated flows matter more. Thus, this approach consists of finding tariffs $\boldsymbol{ET}$ and $\boldsymbol{XT}$ that solve the following least squares minimization problem:

$$\min \sum_{i,j} F_i^{\mathrm{ET}} F_j^{\mathrm{XT}} (ET_i + XT_j - \mathcal{C}_{ij})^2. \tag{5}$$

In Bermúdez et al. (2013) the authors showed that this minimization problem has an infinite number of solutions. More importantly, they showed that, for each possible entry-exit split, there is a unique solution delivering precisely that split. This paper goes one step beyond, obtaining explicit formulas for these solutions. In particular, in Appendix A it is shown that the unique solution delivering a 50-50 split leads to tariffs given, for each entry point i and each exit point j, by

$$ET_i^{\mathrm{L}} = \bar{C}_i^{\mathrm{ET}} - \frac{\hat{C}}{2w^2} \quad \text{and} \quad XT_j^{\mathrm{L}} = \bar{C}_j^{\mathrm{XT}} - \frac{\hat{C}}{2w^2}.$$

It is easy to see that, so defined, the collected revenue is $\frac{\hat{C}}{w}$, as desired.

Interestingly, because of the explicit formulas presented in this section, the least squares methodology is not more computationally demanding than others: the solutions of the underlying optimization problem can be easily computed.

4.2.1. Weighted least squares with multiplicative adjustment

From Eq. (1) in Section 3.2.1 we get, for each entry point i and each exit point j,

$$ET_i^{\mathrm{L}_m} = \frac{2s}{s+1}(\bar{C}_i^{\mathrm{ET}} - \frac{\hat{C}}{2w^2}) \quad \text{and} \quad XT_j^{\mathrm{L}_m} = \frac{2}{s+1}(\bar{C}_i^{\mathrm{XT}} - \frac{\hat{C}}{2w^2}). \tag{6}$$

The main problem of the multiplicative adjustment is that the resulting tariffs do not correspond to any solution of the least squares minimization problem in Eq (5). This issue disappears when the adjustment is additive.

4.2.2. Weighted least squares with additive adjustment

From Eq. (2) in Section 3.2.2 we get, for each entry point i and each exit point j,

$$ET_i^{\mathrm{L}_a} = \bar{C}_i^{\mathrm{ET}} - \frac{1}{s+1}\frac{\hat{C}}{w^2} \quad \text{and} \quad XT_j^{\mathrm{L}_a} = \bar{C}_j^{\mathrm{XT}} - \frac{s}{s+1}\frac{\hat{C}}{w^2}. \tag{7}$$

Importantly, this is the unique methodology with the following property. For each split parameter s, the resulting tariffs are the unique ones that are an optimal solution to the minimization problem in Eq. (5) and that deliver split s.

4.3. Postage stamp methodology

Because of its role in the following section, the postage stamp methodology is now defined. The idea of this approach is to treat all points equally, regardless of their situation in the network. The resulting tariffs, despite being perfectly non-discriminatory, are not cost-reflective at all. When the goal is a 50-50 split, this tariffs are given, for each entry point i and each exit point j, by

$$ET_i^{\mathrm{P}} = \frac{\hat{C}}{2w^2} \quad \text{and} \quad XT_j^{\mathrm{P}} = \frac{\hat{C}}{2w^2}.$$

Now, following the approach in sections 3.2.1 and 3.2.2, it is easy to check that additive and multiplicative adjustments coincide for the postage stamp methodology. The resulting tariffs are given, for each entry point i and each exit point j, by

$$ET_i^{\mathrm{P}_m} = ET_i^{\mathrm{P}_a} = \frac{s}{s+1}\frac{\hat{C}}{w^2} \quad \text{and} \quad XT_j^{\mathrm{P}_m} = XT_j^{\mathrm{P}_a} = \frac{1}{s+1}\frac{\hat{C}}{w^2}. \tag{8}$$

4.4. Unweighted methodologies

For the sake of completeness, the unweighted versions of the above methodologies are briefly described below.

For the unweighted version of the capacity-weighted distance approach, it suffices to work with the (unweighted) averages $\bar{\boldsymbol{U}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{U}}^{\mathbf{XT}}$ instead of $\bar{\boldsymbol{C}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{C}}^{\mathbf{XT}}$. The same applies for the computations with different splits.

As far as the least squares approach is concerned, the unweighted version is the one that has received more attention so far in the literature; see, for instance, Deliberata (2006) Alonso et al. (2010), Apolinário et al. (2012) and FG-2013 (under the name *matrix approach* in this last document).[8] For this methodology, the derivations are analogous to those of the weighted version,

[8] We find this name a bit ambiguous since, in general, the capacity-weighted distance approach might use the same cost matrix as starting point.

but working with the following minimization problem:

$$\min \sum_{i,j} (ET_i + XT_j - \mathcal{C}_{ij})^2. \tag{9}$$

4.5. An alternative approach

Hunt (2008, Appendix A) introduced a methodology that is related to the above ones. To some extent, it can be seen as an approach between the capacity-weighted distance and the least squares ones.

One special feature of Hunt's approach is that the entry-exit split is obtained endogenously during the computations. To be more precise, suppose that we have identified all possible multiplicative adjustments of the unweighted version of the capacity-weighted distance approach. Then, Hunt's methodology selects the one that minimizes the unweighted least squares problem in Eq. (9). In Appendix C it is shown that this "optimal" entry-exit split is given by $s^u = \frac{\alpha^u}{1-\alpha^u}$, where[9]

$$\alpha^u = \frac{\sum_{i=1}^n \sum_{j=1}^m (\mathcal{C}_{ij} - \bar{U}_j^{\mathrm{XT}})(\bar{U}_i^{\mathrm{ET}} - \bar{U}_j^{\mathrm{XT}})}{\sum_{i=1}^n \sum_{j=1}^m (\bar{U}_i^{\mathrm{ET}} - \bar{U}_j^{\mathrm{XT}})^2}.$$

Following the motivation discussed for the other methodologies, one could argue that it would be preferable to define a weighted version of Hunt's approach. First, one would identify all the possible multiplicative adjustments of the capacity-weighted distance approach. Then, one would take the one that minimizes the weighted least squares problem. In Appendix C it is shown that this new "optimal" entry-exit split is given by $s^w = \frac{\alpha^w}{1-\alpha^w}$, where

$$\alpha^w = \frac{\sum_{i=1}^n \sum_{j=1}^m F_i^{\mathrm{ET}} F_j^{\mathrm{XT}} (\mathcal{C}_{ij} - \bar{C}_j^{\mathrm{XT}})(\bar{C}_i^{\mathrm{ET}} - \bar{C}_j^{\mathrm{XT}})}{\sum_{i=1}^n \sum_{j=1}^m F_i^{\mathrm{ET}} F_j^{\mathrm{XT}} (\bar{C}_i^{\mathrm{ET}} - \bar{C}_j^{\mathrm{XT}})^2}.$$

One of the merits of this approach lies in the fact that it determines the entry-exit split following an objective criterion. Yet, it is worth emphasizing that, both in the weighted version and in the unweighted one, the resulting

[9] Actually, Appendix C contains only the derivations for the weighted version of Hunt's approach, with the ones for the unweighted version being analogous.

tariffs are not a solution of the corresponding least squares minimization problem, namely (5) and (9). Indeed, as mentioned in Section 4.2, the unique solution of such minimization problem delivering the identified split is given by the corresponding (weighted/unweighted) least squares tariffs with additive adjustment.

4.6. Summary of the strengths and weaknesses of the methodologies

It is interesting to assess the behavior of the methodologies presented in this section with respect to the principles formulated by the European Commission: transparency, cost-reflectivity, non-discrimination, providing appropriate return on investment,...

Clearly, the postage stamp methodology is completely transparent, but it lacks any kind of cost-reflectivity. The capacity-weighted methodology is also transparent. Moreover, as long as the matrix $\mathcal{C}$ correctly reflects the transmission costs between the different nodes in the network, this methodology should be cost-reflective. Similar considerations apply to the weighted least squares methodology and to Hunt's approach. Certainly, since these last two approaches to solve an optimization problem to compute the tariffs, one may argue that some transparency is lost. Yet, the explicit formulas given in the present paper definitely address this issue. Hunt's methodology has the special feature of endogenously determining the entry-exit split. However, it is not easy to assess whether this should be seen as a strength or a weakness. First, as it is argued above, it is not clear that the resulting tariffs are solving the right optimization problem. Second, although the endogenous choice of the split may prevent arbitrary decisions from the regulating bodies, there is some loss of transparency since there is no intuitive justification for the resulting split. In this respect, the document NC-2016 specifies that any country applying an entry-exit split different from 50-50 has to appropriately justify the chosen one.

On the other hand, the use of unweighted methodologies may lead to discriminatory tariffs. Since the final tariffs crucially depend on the network representation, there is room for representations that favor certain nodes (see the tariffs for nodes E1 and E3 in the example of Section 6).

A primary concern in past literature has been the potential complexity associated with the computation of least square tariffs, since it requires to solve an optimization problem with infinitely many solutions. This should not be seen as a weakness anymore, since this paper provides explicit formulas to easily compute all methodologies (delivering unique tariffs for each split parameter that may be selected).

5. Connections between the different methodologies

Consider the two versions of the capacity-weighted distance approach and the two versions of the weighted least squares approach. It can be easily seen from the expressions obtained in the previous section that all of them are are affine transformations of vectors $\bar{\boldsymbol{C}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{C}}^{\mathbf{XT}}$. Therefore, they are also affine transformations of one another. More precisely, given two vectors of entry (or exit) tariffs $\boldsymbol{X}$ and $\boldsymbol{Y}$ associated with two of the above methodologies, there are constants $a > 0$ and b such that

$$\boldsymbol{Y} = a\boldsymbol{X} + b.$$

In particular, this implies that the two tariff vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ have correlation one. Thus, the main difference between the methodologies lies on the dispersion of the tariffs, as measured by the multiplicative factor a.

Suppose that attention is restricted to the entry tariffs and take as reference those obtained with the capacity-weighted distance approach with multiplicative adjustment, $\boldsymbol{ET}^{\mathbf{C_m}}$. Then, by relying on Eqs.(3)-(7), it is easy to see that, for each entry point i,

$$\begin{aligned} ET_i^{\mathrm{C}_a} &= a^{\mathrm{C}_a} ET_i^{\mathrm{C}_m} + b^{\mathrm{C}_a} = \frac{s+1}{2s} ET_i^{\mathrm{C}_m} + \frac{s-1}{s+1}\frac{\hat{C}}{2w^2}. \\ ET_i^{\mathrm{L}_m} &= a^{\mathrm{L}_m} ET_i^{\mathrm{C}_m} + b^{\mathrm{L}_m} = 2ET_i^{\mathrm{C}_m} - \frac{s}{s+1}\frac{\hat{C}}{w^2}, \quad \text{and} \\ ET_i^{\mathrm{L}_a} &= a^{\mathrm{L}_a} ET_i^{\mathrm{C}_m} + b^{\mathrm{L}_a} = \frac{s+1}{s} ET_i^{\mathrm{C}_m} - \frac{1}{s+1}\frac{\hat{C}}{w^2}, \end{aligned} \tag{10}$$

Similar expressions can be obtained in a straightforward manner for the exit tariffs. Section 5.1 below goes a bit further in exploring the above connections and shows that, if a natural adjustment is made, then all four methodologies coincide.

5.1. Dispersion Control

The FG-2013 document acknowledges that “The setting of tariffs involves certain trade-offs. In order to address those trade-offs, National Regulatory Authorities may decide to adjust methodologies and associated initial tariffs at national level, via secondary adjustments…”. These secondary adjustments, most of which are also considered in NC-2016, include rescaling (multiplicative and additive), equalization and the so called benchmarking. A related secondary adjustment, called *dispersion control*, is developed in Bermúdez et al. (2013). The goal of this adjustment is to mitigate the effects that a high dispersion between tariffs might have in the market, since it facilitates smoother transitions between tariff methodologies.

For the sake of exposition the analysis is developed for the entry tariffs, since the analysis for the exit ones is completely analogous. Suppose that some measure of the dispersion associated to a vector of tariffs has been chosen and that one wants to control it. For instance, take as dispersion measure the ratio between the highest and the lowest entry tariffs. Then, the dispersion control introduced in Bermúdez et al. (2013) consists of taking a combination of the postage stamp tariffs (zero dispersion) and the tariffs at hand. By doing so, dispersion can be brought down to the desired level.

More formally, suppose that there are a vector $\boldsymbol{X}$ representing some entry tariffs and a constant P representing the corresponding postage stamp tariffs. Let $\hat{x} = \max_i X_i$ and $\check{x} = \min_i X_i$. Suppose that we want the ratio between the highest and the lowest entry tariff equals some number $k > 0$. Then, the dispersion control adjustment consists of finding λ such that

$$\lambda P + (1-\lambda)\hat{x} = k\Big(\lambda P + (1-\lambda)\check{x}\Big).$$

Solving for λ we get

$$\lambda_k = \frac{\hat{x} - k\check{x}}{P(k-1) + \hat{x} - k\check{x}}. \tag{11}$$

Then, the adjusted tariffs after dispersion control would be obtained, for each entry point i, as

$$X_i^{D,k} = \lambda_k P + (1 - \lambda_k) X_i.$$

From the above expression it is easy to see that, as long as the initial postage stamp tariffs and the tariffs in vector $\boldsymbol{X}$ collect the same revenue, the resulting $\boldsymbol{X}^{D,k}$ tariffs will also do (see Proposition 1 in Appendix B). Therefore, the dispersion control can be applied after a given entry-exit split has already been imposed, since it will deliver tariffs with the same split.

Next, it is shown that, if the same level of dispersion is imposed to the methodologies discussed in Section 5, then all of them coincide. Further, in Appendix B, where the analysis is developed more formally, it is also shown that this coincidence result is essentially independent of the chosen dispersion measure.

First, take two vectors of tariffs $\boldsymbol{X}$ and $\boldsymbol{Y}$ such that there are constants $a > 0$ and b satisfying

$$\boldsymbol{Y} = a\boldsymbol{X} + b.$$

Suppose now that the dispersion control is applied to these tariffs with the same level k. By Eq. (11) we get (provided that $\hat{x} \neq \check{x}$)

$$\lambda_k^x = \frac{\hat{x} - k\check{x}}{P(k-1) + \hat{x} - k\check{x}} \quad \text{and} \quad \lambda_k^y = \frac{\hat{y} - k\check{y}}{P(k-1) + \hat{y} - k\check{y}}.$$

Therefore, given two specific tariffs x and $y = ax + b$, the corresponding adjusted tariffs would be

$$x^{D,k} = \lambda_k^x P + (1 - \lambda_k^x) x \quad \text{and} \quad y^{D,k} = \lambda_k^y P + (1 - \lambda_k^y) y.$$

Subtracting these adjusted tariffs, after some algebra we get

$$x^{D,k} - y^{D,k} = \frac{\big(-1 + k\big) P \big(b + (a-1)P\big)\big(x - kx - \hat{x} + k\check{x}\big)}{\big((-1+k)P + \hat{x} - k\check{x}\big)\big(b - bk + (-1+k)P + a(\hat{x} - k\check{x})\big)}.$$

Clearly, if $b = (1-a)P$, then the tariffs $x^{D,k}$ and $y^{D,k}$ are equal.[10] Recall that, by Eq. (8), $ET_i^{\mathrm{P}_a} = ET_i^{\mathrm{P}_m} = \frac{s}{s+1}\frac{\hat{C}}{w^2} = P$. Now, consider the tariffs $\boldsymbol{ET}^{\mathbf{C_a}}$. By Eq. (10), it holds that $ET_i^{\mathrm{C}_a} = a^{\mathrm{C}_a} ET_i^{\mathrm{C}_m} + b^{\mathrm{C}_a}$, with $a^{\mathrm{C}_a} = \frac{s+1}{2s}$ and $b^{\mathrm{C}_a} = \frac{s-1}{s+1}\frac{\hat{C}}{2w^2}$. Hence, for the connection between $\boldsymbol{ET}^{\mathbf{C_a}}$ and $\boldsymbol{ET}^{\mathbf{C_m}}$

$$(1 - a^{\mathrm{C}_a})P = (1 - \frac{s+1}{2s})\frac{s}{s+1}\frac{\hat{C}}{w^2} = \frac{s-1}{s+1}\frac{\hat{C}}{2w^2} = b^{\mathrm{C}_a},$$

and the condition $b = (1-a)P$ holds. Analogously, $b = (1-a)P$ also holds for the other two affine transformations in Eq. (10):

$$\begin{aligned}
\boldsymbol{ET}^{\mathbf{L_m}} \quad &\rightarrow \quad (1 - a^{\mathrm{L}_m})P = (1-2)\frac{s}{s+1}\frac{\hat{C}}{w^2} = -\frac{s}{s+1}\frac{\hat{C}}{w^2} = b^{\mathrm{L}_m}, \text{ and} \\
\boldsymbol{ET}^{\mathbf{L_a}} \quad &\rightarrow \quad (1 - a^{\mathrm{L}_a})P = (1 - \frac{s+1}{s})\frac{s}{s+1}\frac{\hat{C}}{w^2} = -\frac{1}{s+1}\frac{\hat{C}}{w^2} = b^{\mathrm{L}_a}.
\end{aligned}$$

Therefore, the above equalities imply that, after the dispersion control adjustment, the tariffs obtained from $\boldsymbol{ET}^{\mathbf{C_a}}$, $\boldsymbol{ET}^{\mathbf{L_m}}$, and $\boldsymbol{ET}^{\mathbf{L_a}}$ coincide with those obtained from $\boldsymbol{ET}^{\mathbf{C_m}}$. Thus, if such a secondary adjustment is imposed, the four methodologies lead to exactly the same tariffs.

Wrapping up, it has been shown that, as long as the same dispersion control is applied to all the entry tariffs (same k) and the same dispersion control is applied to all the exit tariffs (same $\bar{k}$, possibly different from k), the two weighted versions of the capacity-weighted methodology and the two weighted versions of the least squares methodology coincide. It is also worth noting that a similar conclusion would be reached by developing the same analysis for the unweighted methodologies.

6. Illustration and Discussion

This section is devoted to illustrate the methodologies discussed in this paper by studying them on a simple example. Throughout this section

[10] Another sufficient condition is $k = 1$, in which case we would have $\lambda_k^x = \lambda_k^y = 1$ and the final tariffs would coincide with the postage stamp tariffs, *i.e.*, no dispersion at all.

CW-M and CW-A are used to refer to the multiplicative and additive versions of the capacity-weighted distance approach. Similarly, WLS-M and WLS-A are used for the weighted least squares approach. The tariffs obtained via Hunt's weighted methodology are called HUNT-W. The corresponding unweighted versions are named CU-M, CU-A, ULS-M, ULS-A, and HUNT-U. Finally, PT is used to refer to postal tariffs.

Consider the network and flow configuration depicted in Figure 3. There are three entry points and five exit points. A very symmetric network has been chosen because it helps to show more clearly the fundamental differences between weighted and unweighted methodologies. The same differences can also arise in networks without symmetries.

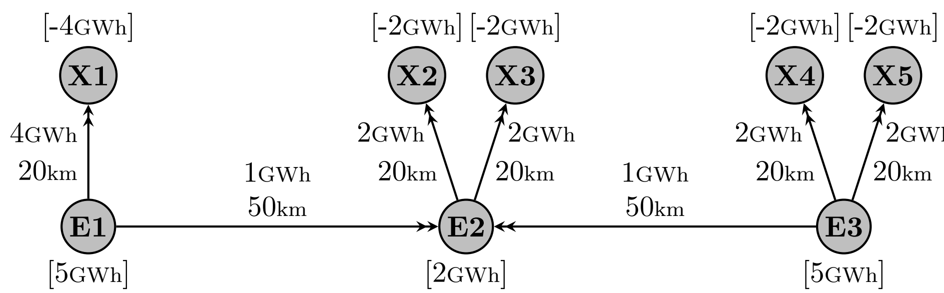

Figure 3: The network and flow configuration.

In order to apply the different tariff methodologies we start by obtaining the cost or distance matrix $\mathcal{C}_{3\times5}$. To do so, when computing the distance between a pair of points, we take into account whether or not pipes are used in the direction of prevalent flows or backhaul. Thus, the associated costs or distances are adjusted (rescaled) using a *backhaul parameter* (see, for instance, Alonso et al. (2010)). In this case, the entries of $\mathcal{C}$ are computed with backhaul parameter 0.1. More precisely, $\mathcal{C}_{ij}$ is computed as the length of the shortest path between entry point i and exit point j, where the lengths of pipes used in backhaul are multiplied by 0.1. Then, the following costs are obtained:

	X1	**X2**	**X3**	**X4**	**X5**
E1	20	70	70	75	75
E2	25	20	20	25	25
E3	75	70	70	20	20

The vectors of entry and exit flows are, respectively, $\boldsymbol{F}^{\mathbf{ET}} = (5, 2, 5)$ and $\boldsymbol{F}^{\mathbf{XT}} = (4, 2, 2, 2, 2)$. The entry-exit split is set to 75-25 ($s = 3$) and the revenue to be collected to $R = \frac{\hat{C}}{w} \approx 596.67$.

Before moving on, note the following characteristics of the network in Figure 3:

(i) Exit points X2 and X3 are completely symmetric, so one should expect to get the same tariffs for them.

(ii) Similarly, by looking at matrix $\mathcal{C}$ one can see that nodes X1, X4 and X5 are essentially symmetric as well.

(iii) Finally, entry points E1 and E3 are also essentially symmetric. To see why, just note that the distance from X4 to any other node in the network coincides with the corresponding distance from X5. Thus, if nodes X4 and X5 are replaced with a single node X6 with demand 4 (the sum of the individual demands of X4 and X5), the resulting network can be seen as equivalent to the original one (after all, depicting X4 and X5 as independent nodes is just a matter of network representation). Yet, in this new network E1 and E3 would be completely symmetric. Therefore, one should expect that E1 and E3 also get the same tariffs.

Table 1 presents the tariffs obtained after applying the formulas in Section 4. From this table it is easy to check that the correlation between the weighted entry tariffs is one and that it is also one between the unweighted entry tariffs (with the same observation being true for the exit tariffs). As mentioned earlier, this is because the four weighted versions are are affine transformations of vectors $\bar{\boldsymbol{C}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{C}}^{\mathbf{XT}}$. Similarly, the unweighted versions are affine transformations of vectors $\bar{\boldsymbol{U}}^{\mathbf{ET}}$ and $\bar{\boldsymbol{U}}^{\mathbf{XT}}$. As a side comment, it is worth noting that in this example one of the methodologies delivered a negative tariff for point E2. Although this raises no problem from the mathematical point of view, one might argue that tariffs should not be negative. Indeed, most methodologies include the specification of secondary adjustments that can correct this kind of issues.

	E1	**E2**	**E3**	**X1**	**X2**	**X3**	**X4**	**X5**
CW-A	39.93	24.1	39.93	9.44	18.40	18.40	9.44	9.44
CW-M	41.25	17.50	41.25	10.94	15.42	15.42	10.94	10.94
WLS-A	45.21	−2.29	45.21	9.44	18.40	18.40	9.44	9.44
WLS-M	42.57	10.90	42.57	6.46	24.38	24.38	6.46	6.46
HUNT-W	36.37	15.43	36.37	14.82	20.89	20.89	14.82	14.82
CU-A	45.53	16.55	37.35	11.24	14.81	14.81	11.24	11.24
CU-M	45.41	16.85	37.35	11.19	14.92	14.92	11.19	11.19
ULS-A	51.92	0.44	37.40	9.89	17.50	17.50	9.89	9.89
ULS-M	48.31	9.55	37.37	8.01	21.27	21.27	8.01	8.01
HUNT-U	52.27	19.39	43.00	6.12	8.15	8.15	6.12	6.12
PT	37.29	37.29	37.29	12.43	12.43	12.43	12.43	12.43

Table 1: Entry-exit tariffs with a 75-25 split ($s = 3$), except for Hunt's methodology, which endogenously obtains $s \approx 1.95$ for the weighted version and $s \approx 6.32$ for the unweighted one.

More importantly, note that all the weighted methodologies respect the symmetries described above. The unweighted versions, however, fail to recognize the symmetry between nodes E1 and E3. Therefore, the unweighted methodologies are sensitive to different representations of the same underlying network. Indeed, it can be seen that all of them deliver a larger tariff for E1 than for E3. Somehow E1 is being "punished" for having to go twice to distant exit points (X4 and X5) whereas E3 only has to go once to a distant exit point (X1). Clearly, if flows are taken into account, both E1 and E3 have to carry the same number of units, four, to distant nodes.

Finally, suppose that we want to impose dispersion control in such a way that the largest entry tariff is exactly twice as large as the smallest one ($k_{\text{entry}} = 2$) and the largest exit tariff is 50% larger than the smallest one

($k_{\text{exit}} = 1.5$).[11] Then, one would obtain the values for λ depicted in Table 2 and the tariffs in Table 3.

	λ entry ($k_{\text{entry}} = 2$)	**λ exit** ($k_{\text{exit}} = 1.5$)
CW-A	−0.28	0.41
CW-M	0.14	−0.19
WLS-A	0.57	0.41
WLS-M	0.36	0.7
HUNT-W	0.14	−0.19
CU-A	0.25	−0.49
CU-M	0.24	−0.43
ULS-A	0.58	0.3
ULS-M	0.44	0.6
HUNT-U	0.24	−0.43

Table 2: Dispersion control coefficients for the different methodologies.

	E1	**E2**	**E3**	**X1**	**X2**	**X3**	**X4**	**X5**
CW-A	40.68	20.34	40.68	10.65	15.98	15.98	10.65	10.65
CW-M	40.68	20.34	40.68	10.65	15.98	15.98	10.65	10.65
WLS-A	40.68	20.34	40.68	10.65	15.98	15.98	10.65	10.65
WLS-M	40.68	20.34	40.68	10.65	15.98	15.98	10.65	10.65
HUNT-W	35.87	17.94	35.87	14.44	21.65	21.65	14.44	14.44
CU-A	43.47	21.73	37.34	10.65	15.98	15.98	10.65	10.65
CU-M	43.47	21.73	37.34	10.65	15.98	15.98	10.65	10.65
ULS-A	43.47	21.73	37.34	10.65	15.98	15.98	10.65	10.65
ULS-M	43.47	21.73	37.34	10.65	15.98	15.98	10.65	10.65
HUNT-U	50.04	25.02	42.98	5.82	8.74	8.74	5.82	5.82

Table 3: Entry-exit tariffs in Table 1 after applying dispersion control with $k = 2$ for the entry tariffs and $k = 1.5$ for the exit tariffs.

As already expected from the results in the previous section, the methodologies CW-A, CW-M, WLS-A and WLS-M deliver exactly the same tariffs (the same being true for the unweighted versions of these tariffs). Since the

[11] There is nothing special in these choices of k_{entry} and k_{exit}, they are just two arbitrary values taken for the sake of illustration.

split endogenously obtained with Hunt's methodology does not coincide with the split used for the other methodologies, Hunt's approach delivers different tariffs. Interestingly, note that the split obtained by this methodology can drastically change depending on whether or not weights are considered. In this example it delivers, approximately, 66-34 with weights and 86-14 without them. In particular, weighted exit tariffs are more than two times larger than the unweighted ones.

Note that, concerning the way in which tariffs respect the symmetries in the underlying network, nothing changes after applying dispersion control.

Finally, note that some of the coefficients in Table 2 are negative, which implies that the original tariffs had a smaller dispersion than the target one. Consequently, the dispersion of those tariffs has increased after applying the dispersion control. Therefore, if the target dispersion is just meant to set an upper bound on the admissible dispersion, one should only adjust the tariffs when this upper bound was exceeded by the achieved tariffs. Yet, if it is applied in this way, the resulting tariffs of the different methodologies might not coincide: in such a case, for a given target dispersion, the dispersion control might only affect some of the methodologies.

6.1. Comparison with other methodologies

In order to get more perspective on the previous analysis, it would be interesting to compare the entry-exit tariffs discussed so far with other approaches such as point to point methodologies. To be able to make this comparison, one first needs to compute the costs of sending one unit of flow from each entry point to each exit point under the entry-exit tariffs. A thorough comparison, which would require to consider a wide number of alternative methodologies and to compare them over a wide number of networks, is beyond the scope of this paper. As an initial step in this direction, we provide below an illustration using the network in Figure 3 and including the tariffs associated to two additional methodologies: the *Long Run Marginal Cost Methodology* (LRMC) as discussed in the report of the National Grid (2011) and the *First Best Mechanism* (FB), following the nodal pricing approach in David and Percebois (2002).[12]

	E1-X1	E1-X2	E1-X4	E2-X1	E2-X2	E2-X4	E3-X1	E3-X2	E3-X4
CW-A	49.37	58.33	49.37	33.54	42.50	33.54	49.37	58.33	49.37
CW-M	52.19	56.67	52.19	28.44	32.92	28.44	52.19	56.67	52.19
WLS-A	54.65	63.61	54.65	7.15	16.11	7.15	54.65	63.61	54.65
WLS-M	49.03	66.95	49.03	17.36	35.28	17.36	49.03	66.95	49.03
PT	49.72	49.72	49.72	49.72	49.72	49.72	49.72	49.72	49.72
LRMC	44.75	82.04	44.75	0.00	37.29	0.00	44.75	82.04	44.75
FB	94.52	38.40	23.63	79.75	23.63	8.86	94.52	38.40	23.63

Table 4: Comparison of the resulting point to point tariffs.

The resulting point to point tariffs are depicted in Table 4.[13] For the sake of comparison, all tariffs are rescaled so that they collect the same revenue (the tariffs associated to X3 and X5 have been omitted since they coincide with those associated to X2 and X4, respectively, because of the network symmetries).

There are important differences between the methodologies. First, the tariffs based on average costs discussed in the previous sections are quite similar to each other. On the other hand, one of the main ideas of the marginal costs' approach of LRMC is to provide signals for network expansions. This is done by taking into account how additional flows between entry and exit nodes might increase or reduce potential congestion issues. Thus, sending additional gas to X1 and X4 is cheaper than doing it to X2, because sending gas from E2 to X1 and X4 would reduce potential congestion issues in the pipes E1-E2 and E3-E2.

Finally, it is worth noting that, according to FB tariffs, sending gas to X1

[12]Ideally, we would have liked to include in the analysis the classic Atlantic Seaboard Formula (see, for instance, Wellisz (1963)), but its computation requires the specification of a good number of additional elements (which play no role for the other methodologies) and the comparison would crucially depend on the specific choices made for them.

[13]For the computation of the LRMC tariffs, we took as virtual point the node E2, the natural choice given that it is the center of the network. Since the first best mechanism described in David and Percebois (2002) depends on the pressure loss associated with the flow transmission between nodes, the diameter of the pipes should be specified to compute it. In our analysis we just assume that all pipes have the same diameter, so that this choice does not affect the resulting tariffs.

would be notably more expensive than sending gas to any other node. The reason for this is that the first best methodology as described in David and Percebois (2002) has as its main cost driver the pressure loss associated with the different gas paths in the network. Since pressure loss is proportional to the length of the pipe and the squared flow (we are assuming that all pipes have the same diameter), the fact that pipe E1-X1 carries twice as much flow as any other pipe heavily penalizes node X1 in this example.

The numbers in Table 4 as well as the above discussion suggest that regulators and policy makers, when choosing a tariff methodology, should carefully consider what are the underlying cost drivers and the ensuing network signals.

7. Conclusions

The focus of this paper has been on two of the main tariff methodologies discussed in the documents published since 2013 by ACER, ENTSOG and the European Commission. These methodologies are intended to be the basis for more integrated and harmonized transmission tariff structures within the European gas networks. As it was already mentioned in the introduction, the main insights obtained from the formal analysis in this paper were presented to ENTSOG in early 2014. Then, to some extent, the changes observed in NC-2016 regarding the elimination of the least squares methodology and the unweighted methodologies can be seen as a natural reaction to our recommendations.

A first contribution of the analysis is the development of explicit formulas for the computation of these methodologies. Specially relevant are the simple formulas obtained for the least squares approach (matrix approach in FG-2013). This is because the FG-2013 document (Section 3.2.1.1.) says "The choice for or against the matrix methodology, or the virtual point methodology, relative to the capacity-weighted distance methodologies, shall consider both the drawback of necessary network representation simplifications and the benefit in cost-reflectivity, as compared to the capacity-weighted distance approach." Based on the obtained formulas, the capacity-weighted distance approach and the matrix approach require equally simple computations, so we see no reason for network simplifications regardless of which of the two is to be applied. Further, since the resulting tariffs are very similar, there is neither benefit nor lost of cost-reflectivity depending on the chosen methodology.

Also important for its potential policy implications is the main result of this paper, which establishes that capacity-weighted and least squares methodologies lead to very similar tariffs. This suggests that having both of them as available methodologies is a redundancy that may just make harmonization more difficult and harm transparency. Since the tariffs obtained by these two methodologies have correlation one, the only difference comes from their dispersion. In particular, if the entry and exit points are ranked according to their tariffs, then both methodologies lead to the same rankings. More formally, it is shown that if a natural dispersion control is applied to obtain the final tariffs, then the two methodologies yield exactly the same tariffs.[14]

Importantly, dispersion control has other benefits that are of potential interest for regulators. First, since this secondary adjustment allows to control the ratio between the largest and the smallest tariff, it provides a natural tool to "implement mitigating measures" that ensure "price stability", as suggested by FG-2013. Second, since this control is obtained by a combination of the methodology-specific tariffs with the flat (nonnegative) postage stamp tariffs, it provides a tool to ensure nonnegativity of the final prices.

This paper raises a relevant issue that, so far, has received virtually no attention in most of the documents regarding tariffs. When formally defining a tariff methodology, there are normally several steps where one has to take averages involving entry or exit points. One can choose plain or weighted averages, with the latter ones taking into account the capacities of the different

[14]Interestingly, recall that one can perform a secondary adjustment on the tariffs to obtained a target entry-exit split. The above result holds both for additive or multiplicative adjustments. Moreover, it does not rely on the specific choice of the dispersion measure.

points. Several examples in the paper illustrate that this may have a high impact in the resulting tariffs. Therefore, this aspect of tariff computation should not be overlooked when designing a methodology. Importantly, the arguments presented in the text suggest that weighted methodologies are more suitable to capture relevant features of the underlying transmission network.

There are quite a few directions for future research that might be worth exploring. First, it would be interesting to expand the analysis in Section 6.1 and make a thorough comparison of different methodologies and their behavior on different networks. Second, the normative analysis developed in this paper might be applied to other energy networks both inside and outside the European Union. For instance, it could foster new cooperations between the European Union and the Gulf Cooperation Council, in the spirit of the projects discussed in Doukas et al. (2012, 2013). Finally, it is worth noting that the analysis in this paper just provides what one might call a definite "static" picture of these methodologies. In this sense, it should be seen as the basis for the literature that studies the implications of the chosen tariffs on "dynamic" aspects such as the strategic behavior of the actors of the energy markets. In this literature, game theoretical models are developed and questions such as optimal pricing, social welfare, and incentive compatibility are analyzed (see, for instance, Cremer et al. (2003), Gasmi and Oviedo (2010), Brandão et al. (2014), and references therein).

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Appendix A. Obtaining an explicit expression for the weighted least squares tariffs

For the analysis in this section we use vector notation. In particular, vectors are represented by capital bold face letters and matrices by capital calligraphic ones. We present below the notation used throughout this appendix.

Consider a gas transport network with n entry points and m exit points. We denote by $\boldsymbol{X} = (X_1, \ldots, X_n)^t \in \mathbb{R}^n$ and $\boldsymbol{Y} = (Y_1, \ldots, Y_m)^t \in \mathbb{R}^m$ the vector of entry and exit tariffs, respectively. The cost matrix is $\mathcal{C} \in \mathcal{M}_{n\times m}$. Also, we define the vectors $\boldsymbol{N} = (1, \ldots, 1)^t \in \mathbb{R}^n$ and $\boldsymbol{M} = (1, \ldots, 1)^t \in \mathbb{R}^m$. Finally, denote by $\mathcal{E} \in \mathcal{M}_{n\times n}$ the diagonal matrix such that $\mathcal{E}_{ii} = E_i$, the strictly positive flow through entry point i. Similarly, $\mathcal{F} \in \mathcal{M}_{m\times m}$ denotes the diagonal matrix such that $\mathcal{F}_{jj} = F_j$, the strictly positive flow through exit point j. We assume that the flows in the system are balanced, namely,

$$w := \sum_{i=1}^{n} E_i = \sum_{j=1}^{m} F_j, \quad \text{or, equivalently,} \quad w := \boldsymbol{N}^t \mathcal{E} \boldsymbol{N} = \boldsymbol{M}^t \mathcal{F} \boldsymbol{M}.$$

Also, recall that we assume that the total revenue to collect is given by

$$R = \frac{\hat{C}}{w}, \quad \text{where} \quad \hat{C} := \sum_{i=1}^{n} \sum_{j=1}^{m} E_i F_j \mathcal{C}_{ij} = \boldsymbol{N}^t \mathcal{E} \mathcal{C} \mathcal{F} \boldsymbol{M} = \boldsymbol{M}^t \mathcal{F} \mathcal{C}^t \mathcal{E} \boldsymbol{N}.$$

Then, the *capacity-weighted average* of the rows and columns of matrix $\mathcal{C}$ are given by

$$\bar{\boldsymbol{C}}^{\mathbf{ET}} = \frac{\mathcal{C} \mathcal{F} \boldsymbol{M}}{\boldsymbol{M}^t \mathcal{F} \boldsymbol{M}} \qquad \text{and} \qquad \bar{\boldsymbol{C}}^{\mathbf{XT}} = \frac{\mathcal{C}^t \mathcal{E} \boldsymbol{N}}{\boldsymbol{N}^t \mathcal{E} \boldsymbol{N}}.$$

We now focus on the tariffs under the *weighted least squares approach.* Rewriting the minimization problem in Eq. (5) using the vector notation, we look for $\boldsymbol{X}$ and $\boldsymbol{Y}$ that minimize the following objective function:

$$\varphi(\boldsymbol{X}, \boldsymbol{Y}) = \left\| \mathcal{E}^{\frac{1}{2}} \left(\boldsymbol{X} \boldsymbol{M}^t + \boldsymbol{N} \boldsymbol{Y}^t - \mathcal{C} \right) \mathcal{F}^{\frac{1}{2}} \right\|_2^2,$$

where the nonnegativity of the flows ensures that the $\mathcal{E}^{\frac{1}{2}}$ and $\mathcal{F}^{\frac{1}{2}}$ matrices are well-defined. Then, the conditions for $\boldsymbol{X}, \boldsymbol{Y}$ to define a critical point of φ are

$$\begin{aligned} D_{\boldsymbol{X}} \varphi(\boldsymbol{X}, \boldsymbol{Y})(\delta \boldsymbol{X}) &= 0, \quad \text{for all } \delta \boldsymbol{X} \in \mathcal{M}_{n \times 1}, \text{ and} \\ D_{\boldsymbol{Y}} \varphi(\boldsymbol{X}, \boldsymbol{Y})(\delta \boldsymbol{Y}) &= 0, \quad \text{for all } \delta \boldsymbol{Y} \in \mathcal{M}_{m \times 1}, \end{aligned}$$

where $D_{\boldsymbol{X}} \varphi(\boldsymbol{X}, \boldsymbol{Y})$ and $D_{\boldsymbol{Y}} \varphi(\boldsymbol{X}, \boldsymbol{Y})$ are the partial derivatives of φ with respect to $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively. After some algebra, the above equations reduce to

$$\begin{aligned} \left(\mathcal{E}^{\frac{1}{2}} \left(\boldsymbol{X} \boldsymbol{M}^t + \boldsymbol{N} \boldsymbol{Y}^t - \mathcal{C} \right) \mathcal{F}^{\frac{1}{2}} \right) &\cdot \left(\mathcal{E}^{\frac{1}{2}} \delta \boldsymbol{X} \boldsymbol{M}^t \mathcal{F}^{\frac{1}{2}} \right) = 0, \text{ and} \\ \left(\mathcal{E}^{\frac{1}{2}} \left(\boldsymbol{X} \boldsymbol{M}^t + \boldsymbol{N} \boldsymbol{Y}^t - \mathcal{C} \right) \mathcal{F}^{\frac{1}{2}} \right) &\cdot \left(\mathcal{E}^{\frac{1}{2}} \boldsymbol{N} \delta \boldsymbol{Y}^t \mathcal{F}^{\frac{1}{2}} \right) = 0, \end{aligned}$$

where the dot denotes the scalar product for matrices, that is, given $\mathcal{U}, \mathcal{V} \in \mathcal{M}_{p \times q}$, then $\mathcal{U} \cdot \mathcal{V} = \sum_{i=1}^{p} \sum_{j=1}^{q} U_{ij} V_{ij}$.

From the last two equations, using the equalities $\mathcal{A} \cdot (\mathcal{B}\mathcal{C}) = (\mathcal{A}\mathcal{C})^t \cdot \mathcal{B} =$

$\left(\mathcal{B}^t\mathcal{A}\right)\cdot\mathcal{C}$, we obtain two new equations:

$$\begin{aligned} \boldsymbol{X}\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}+\boldsymbol{N}\boldsymbol{Y}^t\mathcal{F}\boldsymbol{M} &= \mathcal{C}\mathcal{F}\boldsymbol{M}, \text{ and} \qquad \text{(A.1)}\\ \boldsymbol{N}^t\mathcal{E}\boldsymbol{X}\boldsymbol{M}^t+\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}\boldsymbol{Y}^t &= \boldsymbol{N}^t\mathcal{E}\mathcal{C}. \qquad \text{(A.2)} \end{aligned}$$

Next, from Eq. (A.2) we get

$$\boldsymbol{Y}^t=\frac{\boldsymbol{N}^t\mathcal{E}\mathcal{C}-\boldsymbol{N}^t\mathcal{E}\boldsymbol{X}\boldsymbol{M}^t}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}} \qquad \text{(A.3)}$$

and, finally, replacing in Eq. (A.1) we get

$$\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}\left(\boldsymbol{I}-\frac{\boldsymbol{N}\boldsymbol{N}^t\mathcal{E}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}\right)\boldsymbol{X}=\left(\boldsymbol{I}-\frac{\boldsymbol{N}\boldsymbol{N}^t\mathcal{E}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}\right)\mathcal{C}\mathcal{F}\boldsymbol{M}.$$

Therefore, a particular solution is

$$\boldsymbol{X}=\frac{\mathcal{C}\mathcal{F}\boldsymbol{M}}{\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}}.$$

Furthermore, it is easy to show that

$$\operatorname{Ker}\left(\boldsymbol{I}-\frac{\boldsymbol{N}\boldsymbol{N}^t\mathcal{E}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}\right)=\langle\boldsymbol{N}\rangle,$$

where $\langle\boldsymbol{N}\rangle$ denotes the linear space spanned by vector $\boldsymbol{N}$. This is shown by first proving that it has dimension one and then noting that

$$\left(\boldsymbol{I}-\frac{\boldsymbol{N}\boldsymbol{N}^t\mathcal{E}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}\right)(\alpha\boldsymbol{N})=\alpha\boldsymbol{N}-\alpha\frac{\boldsymbol{N}\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}=\boldsymbol{0}.$$

Therefore, the general solution is given by the following affine space in $\mathbb{R}^n$:

$$\boldsymbol{X}=\frac{\mathcal{C}\mathcal{F}\boldsymbol{M}}{\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}}+\alpha\boldsymbol{N},\ \ \alpha\in\mathbb{R}.$$

Replacing this expression in Eq. (A.3) we get

$$\boldsymbol{Y}=\frac{\mathcal{C}^t\mathcal{E}\boldsymbol{N}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}-\frac{\boldsymbol{M}^t\mathcal{F}\mathcal{C}^t\mathcal{E}\boldsymbol{N}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}}\boldsymbol{M}-\alpha\boldsymbol{M},$$

that is,

$$Y_j = \frac{\sum_{i=1}^n \mathcal{C}_{ij} E_i}{\sum_{i=1}^n E_i} - \frac{\sum_{i=1}^n \sum_{k=1}^m \mathcal{C}_{ik} E_i F_k}{\sum_{i=1}^n E_i \sum_{k=1}^m F_k} - \alpha.$$

Therefore, the general solution of the weighted least squares problem (WLS) is the one-dimensional affine space in $\mathbb{R}^{n+m}$ defined by

$$\left.\begin{array}{rcl} \boldsymbol{X} & = & \bar{\boldsymbol{C}}^{\mathbf{ET}} + \xi \boldsymbol{N} \\ \boldsymbol{Y} & = & \bar{\boldsymbol{C}}^{\mathbf{XT}} - z\boldsymbol{M} - \xi \boldsymbol{M} \end{array}\right\} \quad \xi \in \mathbb{R}, \tag{A.4}$$

where

$$z = \frac{\boldsymbol{M}^t \mathcal{F} \mathcal{C}^t \mathcal{E} \boldsymbol{N}}{\boldsymbol{M}^t \mathcal{F} \boldsymbol{M} \boldsymbol{N}^t \mathcal{E} \boldsymbol{N}} = \frac{\sum_{i=1}^n \sum_{j=1}^m \mathcal{C}_{ij} E_i F_j}{\left(\sum_{i=1}^n E_i\right) \left(\sum_{j=1}^m F_j\right)} = \frac{\hat{C}}{w^2}.$$

Appendix B. Dispersion control and coincidence of tariffs

First recall that the vectors of postage stamp tariffs with an entry-exit split s are

$$\boldsymbol{X}^P = \frac{s}{s+1} \frac{\hat{C}}{w^2} \boldsymbol{N} \quad \text{and} \quad \boldsymbol{Y}^P = \frac{1}{s+1} \frac{\hat{C}}{w^2} \boldsymbol{M}.$$

Recall as well that, for this methodology, additive and multiplicative adjustments lead to the same tariffs.

Now, the goal is to control the dispersion of the entry and/or exit tariffs. For this purpose two parameters, λ_E for the entry tariffs and λ_X for the exit ones, are chosen in order to get a linear combination between the postage stamp tariffs and any of the other tariffs obtained with control of the split:

$$\begin{array}{rcl} \boldsymbol{X}^{\lambda_E} = \lambda_E \boldsymbol{X}^P & + & (1-\lambda_E)\, \boldsymbol{X} \\ \boldsymbol{Y}^{\lambda_X} = \lambda_X \boldsymbol{Y}^P & + & (1-\lambda_X)\, \boldsymbol{Y}. \end{array} \tag{B.1}$$

The values for λ_E and λ_X are chosen in order to satisfy a certain condition related to the dispersion of the corresponding tariffs. We present now a series of straightforward results related to the $\boldsymbol{X}^{\lambda_E}$ and $\boldsymbol{Y}^{\lambda_X}$ tariffs.

Proposition 1. *If the revenue collected by the original tariffs is R, the tariffs $\boldsymbol{X}^{\lambda_E}$ and $\boldsymbol{Y}^{\lambda_X}$ also collect R.*

Proof. Suppose that we have entry and exit tariffs $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively, with an entry-exit split s. By definition of the split we have

$$N^t \mathcal{E} \boldsymbol{X} = \frac{s}{s+1} R \quad \text{and} \quad \boldsymbol{M}^t \mathcal{F} \boldsymbol{Y} = \frac{1}{s+1} R.$$

Now, the revenue collected with the $\boldsymbol{X}^{\lambda_E}$ and $\boldsymbol{Y}^{\lambda_X}$ tariffs can be calculated as $\boldsymbol{N}^t \mathcal{E} \boldsymbol{X}^{\lambda_E} + \boldsymbol{M}^t \mathcal{F} \boldsymbol{Y}^{\lambda_X}$, which reduces to

$$\begin{aligned}
&\boldsymbol{N}^t \mathcal{E} \left(\lambda_E \boldsymbol{X}^P\right) + \boldsymbol{N}^t \mathcal{E} \left(\left(1-\lambda_E\right) \boldsymbol{X}\right) + \boldsymbol{M}^t \mathcal{F} \left(\lambda_X \boldsymbol{Y}^P\right) + \boldsymbol{M}^t \mathcal{F} \left(\left(1-\lambda_X\right) \boldsymbol{Y}\right) \\
&= \lambda_E \frac{s}{s+1} R + \left(1-\lambda_E\right) \frac{s}{s+1} R + \lambda_X \frac{1}{s+1} R + \left(1-\lambda_X\right) \frac{1}{s+1} R \\
&= \frac{s}{s+1} R + \frac{1}{s+1} R = R.
\end{aligned}$$

□

Proposition 2. *The tariffs $\boldsymbol{X}^{\lambda_E}$ and $\boldsymbol{Y}^{\lambda_X}$ preserve the entry-exit split.*

Proof. Suppose that the original tariffs have a given split s, then

$$\begin{aligned}
\frac{\boldsymbol{N}^t \mathcal{E} \boldsymbol{X}^{\lambda_E}}{\boldsymbol{M}^t \mathcal{F} \boldsymbol{Y}^{\lambda_X}} &= \frac{\boldsymbol{N}^t \mathcal{E} \left(\lambda_E \boldsymbol{X}^P\right) + \boldsymbol{N}^t \mathcal{E} \left(\left(1-\lambda_E\right) \boldsymbol{X}\right)}{\boldsymbol{M}^t \mathcal{F} \left(\lambda_X \boldsymbol{Y}^P\right) + \boldsymbol{M}^t \mathcal{F} \left(\left(1-\lambda_X\right) \boldsymbol{Y}\right)} \\
&= \frac{\lambda_E \frac{s}{s+1} R + \left(1-\lambda_E\right) \frac{s}{s+1} R}{\lambda_X \frac{1}{s+1} R + \left(1-\lambda_X\right) \frac{1}{s+1} R} \\
&= \frac{\frac{s}{s+1} R}{\frac{1}{s+1} R} = s.
\end{aligned}$$

□

Proposition 3. *Suppose that the vector of entry tariffs $\boldsymbol{X}$ comes from one of the two versions of the capacity-weighted distance approach or one of the two versions of the weighted least squares approach. Then, regardless, of which one of the four is chosen, the set of entry tariffs $\{\boldsymbol{X}^{\lambda_E},\ \lambda_E \in \mathbb{R}\}$ coincides with the one-dimensional affine space given by*

$$\frac{s}{s+1} \frac{\hat{C}}{w^2} \boldsymbol{N} + \langle \frac{\hat{C}}{w^2} \boldsymbol{N} - \bar{\boldsymbol{C}}^{\boldsymbol{ET}} \rangle.$$

An analogous result holds for exit tariffs.

Proof. To prove the result we just characterize the set $\{\boldsymbol{X}^{\lambda_E},\ \lambda_E \in \mathbb{R}\}$ for the four different methodologies to see that they coincide:

- *Capacity-weighted distance tariffs with additive adjustment:*

$$\lambda_E \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (1-\lambda_E)\left(\frac{\bar{\boldsymbol{C}}^{\mathbf{ET}}}{2} + \frac{s-1}{s+1}\frac{\hat{C}}{2w^2}\boldsymbol{N}\right) =$$

$$= \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad \frac{\lambda_E - 1}{2}\left(\frac{\hat{C}}{w^2}\boldsymbol{N} - \bar{\boldsymbol{C}}^{\mathbf{ET}}\right).$$

- *Weighted least squares tariffs with additive adjustment:*

$$\lambda_E \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (1-\lambda_E)\left(\bar{\boldsymbol{C}}^{\mathbf{ET}} + \frac{1}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N}\right) =$$

$$= \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (\lambda_E - 1)\left(\frac{\hat{C}}{w^2}\boldsymbol{N} - \bar{\boldsymbol{C}}^{\mathbf{ET}}\right).$$

- *Capacity-weighted distance tariffs with multiplicative adjustment:*

$$\lambda_E \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (1-\lambda_E)\,\frac{s}{s+1}\bar{\boldsymbol{C}}^{\mathbf{ET}} =$$

$$= \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (\lambda_E - 1)\,\frac{s}{s+1}\left(\frac{\hat{C}}{w^2}\boldsymbol{N} - \bar{\boldsymbol{C}}^{\mathbf{ET}}\right).$$

- *Weighted least squares tariffs with multiplicative adjustment:*

$$\lambda_E \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (1-\lambda_E)\,\frac{2s}{s+1}\left(\bar{\boldsymbol{C}}^{\mathbf{ET}} - \frac{\hat{C}}{2w^2}\boldsymbol{N}\right) =$$

$$= \frac{s}{s+1}\frac{\hat{C}}{w^2}\boldsymbol{N} \quad + \quad (\lambda_E - 1)\,\frac{2s}{s+1}\left(\frac{\hat{C}}{2w^2}\boldsymbol{N} - \bar{\boldsymbol{C}}^{\mathbf{ET}}\right).\square$$

Corollary 1. *Given a dispersion control that uniquely determines the parameter λ_E, the resulting tariffs for the four methodologies discussed in Proposition 3 coincide. An analogous result holds for exit tariffs.*

Appendix C. Obtaining the entry-exit split via the weighted version of Hunt's approach

The tariffs obtained by the weighted version of Hunt's approach (Hunt 2008, Appendix A) are given by

$$\boldsymbol{X} = \alpha \bar{\boldsymbol{C}}^{\mathbf{ET}} \qquad \text{and} \qquad \boldsymbol{Y} = (1-\alpha)\, \bar{\boldsymbol{C}}^{\mathbf{XT}},$$

where α is chosen to minimize the function

$$\Psi(\alpha) = \varphi\left(\alpha \bar{\boldsymbol{C}}^{\mathbf{ET}}, (1-\alpha)\, \bar{\boldsymbol{C}}^{\mathbf{XT}}\right) = \left\| \mathcal{E}^{\frac{1}{2}} \left(\alpha \bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t + (1-\alpha)\, \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} - \mathcal{C} \right) \mathcal{F}^{\frac{1}{2}} \right\|_2^2 .$$

The first order condition of the minimization problem for α is

$$D_\alpha \Psi(\alpha)(\delta\alpha) = 0, \quad \text{for all } \delta\alpha \in \mathbb{R}.$$

After some algebra we get

$$\begin{aligned} \Psi'(\alpha)\, \delta\alpha &= \mathcal{E}^{\frac{1}{2}} \left(\alpha \bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t + (1-\alpha)\, \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} - \mathcal{C} \right) \mathcal{F}^{\frac{1}{2}} \\ &\cdot \ \mathcal{E}^{\frac{1}{2}} \left(\delta\alpha \bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t - \delta\alpha \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right) \mathcal{F}^{\frac{1}{2}}, \end{aligned}$$

and, therefore,

$$\Psi'(\alpha) = \mathcal{E} \left(\alpha \bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t + (1-\alpha)\, \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} - \mathcal{C} \right) \cdot \left(\bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t - \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right).$$

Solving for $\Psi'(\alpha) = 0$ we get

$$\mathcal{E} \left(\bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t - \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right) \cdot \left(\bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t - \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right) \mathcal{F}\, \alpha = \mathcal{E} \left(\mathcal{C} - \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right) \cdot \left(\bar{\boldsymbol{C}}^{\mathbf{ET}} \boldsymbol{M}^t - \boldsymbol{N} \bar{\boldsymbol{C}}^{\mathbf{XT}^t} \right) \mathcal{F}.$$

Thus,

$$\begin{aligned}
\alpha &= \frac{\mathcal{E}\left(\mathcal{C}-\boldsymbol{N}\bar{\boldsymbol{C}}^{\mathbf{XT}t}\right)\cdot\left(\bar{\boldsymbol{C}}^{\mathbf{ET}}\boldsymbol{M}^t-\boldsymbol{N}\bar{\boldsymbol{C}}^{\mathbf{XT}t}\right)\mathcal{F}}{\mathcal{E}\left(\bar{\boldsymbol{C}}^{\mathbf{ET}}\boldsymbol{M}^t-\boldsymbol{N}\bar{\boldsymbol{C}}^{\mathbf{XT}t}\right)\cdot\left(\bar{\boldsymbol{C}}^{\mathbf{ET}}\boldsymbol{M}^t-\boldsymbol{N}\bar{\boldsymbol{C}}^{\mathbf{XT}t}\right)\mathcal{F}} \\
&= \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}E_iF_j\left(\mathcal{C}_{ij}-\bar{C}_j^{\mathrm{ET}}\right)\left(\bar{C}_i^{\mathrm{ET}}-\bar{C}_j^{\mathrm{ET}}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{m}E_iF_j\left(\bar{C}_i^{\mathrm{ET}}-\bar{C}_j^{\mathrm{ET}}\right)^2}.
\end{aligned}$$

To conclude, just note that the entry-exit split s via the weighted version of Hunt's approach under this α value is

$$s=\frac{\boldsymbol{N}^t\mathcal{E}\boldsymbol{X}}{\boldsymbol{M}^t\mathcal{F}Y}=\frac{\alpha}{1-\alpha}\frac{\frac{\boldsymbol{N}^t\mathcal{E}\mathcal{C}\mathcal{F}\boldsymbol{M}}{\boldsymbol{M}^t\mathcal{F}\boldsymbol{M}}}{\frac{\boldsymbol{M}^t\mathcal{F}\mathcal{C}\mathcal{E}\boldsymbol{N}}{\boldsymbol{N}^t\mathcal{E}\boldsymbol{N}}}=\frac{\alpha}{1-\alpha}\,.$$