Matching and Price Competition: Beyond Symmetric Linear Costs

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Abstract

Bulow and Levin's (2006) "Matching and Price Competition" studies a matching model in which hospitals compete for interns by offering wages. We relax the assumption of symmetric linear costs and compare the pricing equilibrium that results to the firm-optimal competitive equilibrium. With linear and asymmetric costs, competition in the pricing equilibrium may not be localized, but all other qualitative comparisons of Bulow and Levin (2006) hold. With non-linear and symmetric costs workers' average utility in the pricing equilibrium may be higher than in the firm-optimal competitive equilibrium. With asymmetric and non-linear costs, firms need not choose scores from an interval in a pricing equilibrium, which may make competition even less localized.

1 Introduction

Jeremy Bulow and Jonathan Levin (2006), henceforth B&L, investigated a matching model that captures certain features of the National Residents Matching Program (NRMP), the central clearinghouse that assigns medical residents to hospitals. In B&L's model, hospitals of known and varying quality compete for residents of known and varying quality by offering wages. The hospital that offers the highest wage obtains the best resident, the hospital that offers the second-highest wage obtains the second-best resident, etc. Because hospitals commit to wages before the matching is determined, the pricing equilibrium that arises has some of the features of an all-pay auction with complete information. In particular, hospitals employ mixed strategies, so with some probability the matching is inefficient. B&L compare this pricing equilibrium to the hospital-optimal competitive equilibrium, which is the competitive equilibrium with the lowest wages. All competitive equilibria are efficient, matching hospitals and workers assortatively. B&L find that compared to the hospital-optimal competitive equilibrium the pricing equilibrium has lower, more compressed wages and higher hospital profits. They also find that in the pricing equilibrium competition is "localized," in the sense that hospitals compete against hospitals of similar quality.[1]

In this note we investigate the importance of the assumption that hospitals compete by offering wages.[2] As B&L note (pages 653 and 657, and footnote 16), hospitals in fact offer a set of job features, which include wages, reputation, responsibility, work hours, training, quality of facilities, number of senior physicians, etc. This means that the cost of increasing a hospital's attractiveness to residents is not necessarily linear. In addition, hospitals may differ in their cost of providing these different features. For example, a hospital with many senior physicians may be in a better position to attract additional senior physicians; a reputable hospital usually has an advantage over less-known hospitals; a university hospital may find it less costly to provide its residents with research time or training; a rural hospital may incur lower costs than an urban hospital when increasing wages, because of a lower tax rate. The non-linearity and asymmetry in hospitals' costs suggest that B&L's assumption that residents rank hospitals according to expenditures deserves closer inspection.

[1] More precisely, each hospital chooses its wage from an interval, and these intervals overlap for hospitals of similar quality. In addition, for any two hospitals, every wage in the wage interval of the higher-quality hospital is either in the wage interval of the lower-quality hospital or is higher than every wage in the wage interval of the lower-quality hospital.

[2] The relevance of other assumptions in B&L's model has already been studied by Artemov (2008), Kojima (2007), and Nierdele (2007).

This assumption can be separated into two parts. The first is that all hospitals have the same cost of achieving any given level of attractiveness to residents. A hospital's level of attractiveness to residents, which we call "score," is determined by the combination of features the hospital provides. One unit of score has a dollar value of one for residents. B&L assume that all hospitals have the same cost of achieving any given score. The second part of B&L's assumption is that hospitals' technologies of producing score exhibit constant returns to scale, i.e., they are linear and, moreover, the cost of producing score x is x. This means that one dollar of investment by a hospital creates one dollar of value for its residents.

We relax each part of B&L's assumption in turn, and then both parts simultaneously. We find that relaxing symmetry and maintaining linearity does not change B&L's qualitative conclusions regarding the compression of residents' utilities, but can lead to hospital profits and average resident utilities that are equal to those obtained in the hospital-optimal competitive equilibrium. It can also make competition less localized than in B&L's pricing equilibrium.[3] Relaxing linearity changes B&L's qualitative conclusions regarding residents' utilities, even when symmetry is maintained. With non-linear costs, residents may be better off on average compared to the firm-optimal competitive equilibrium. When both symmetry and linearity are relaxed, hospitals may use non-interval bidding strategies.[4] This too implies that competition may be less localized than in B&L's setting.

The matching model we consider here can be thought of as a complete-information all-pay auction with heterogeneous prizes. While similar models have been analyzed when prizes are homogeneous (González-Díaz (2011), Siegel (2009, 2010, 2011)) or players have identical linear costs (B&L, Barut and Kovenock (1998), Xiao (2011)), the combination of heterogeneous prizes and players with asymmetric and/or non-linear costs has not been previously explored. Our findings suggest that existing results may not readily apply to this more general environment.

2 The Model of B&L

There are N firms and N workers. The surplus of firm n from employing worker m is $\Delta_n \cdot m$, where $\Delta_n > 0$ is non-decreasing in n. Firms compete by each offering a non-negative wage. Worker m is matched with the firm that offered the $(N - m + 1)^{\text{th}}$ highest wage. B&L show that in equilibrium each firm chooses the wage it offers by randomizing

[3] Asymmetric linear costs can cause the wage intervals of hospitals of widely varying qualities to overlap.

[4] Interval bidding strategies are a key feature in B&L's analysis.

on an interval of wages, and use this observation and the fact that firms have constant marginal costs to provide an algorithm that constructs the equilibrium. The algorithm proceeds from high wages to low wages, and for every wage identifies every firm's density of offering that wage. The set of "active" firms, whose density of offering that wage is positive, is the maximal set of firms that have not yet exhausted their probability, such that no local equilibrium conditions are violated. The algorithm terminates at wage 0 when all firms except, perhaps, firm 1 have exhausted their probability. Firm 1 chooses wage 0 with its remaining probability. The highest wage need not be specified in advance; rather, the range of wages on which firms compete is identified once the algorithm terminates. This is possible because firms' marginal costs are the same at all wages. The algorithm yields a probability distribution G_n for each firm n, where $G_n(x)$ is the probability that firm n offers a wage less than or equal to x.

3 Firms with Asymmetric Linear Costs

Suppose that instead of offering a wage, each firm offers a "score," which is the monetary value that workers assign to the bundle of attributes associated with working for the firm. As discussed in the introduction, the cost of offering a given score may vary across firms. Suppose that firm n's cost of offering score x is $c_n(x) = \gamma_n x$ (in B&L, $\gamma_n = 1$ for every firm n). To facilitate the comparison to B&L, we restrict attention to costs such that $\frac{\Delta_n}{\gamma_n}$ are non-decreasing in n. This guarantees that competitive equilibria lead to assortative matching.[5]

In a competitive equilibrium in which worker n's price (in units of score) is p_n, firm $n-1$ prefers hiring worker $n-1$ to hiring worker n, so $\gamma_{n-1}p_n - \gamma_{n-1}p_{n-1} \geq \Delta_{n-1}$, or $p_n - p_{n-1} \geq \frac{\Delta_{n-1}}{\gamma_{n-1}}$. In the firm-optimal competitive equilibrium this inequality is an equality, so the firm-optimal competitive equilibrium has prices $p_n = \sum_{k<n} \frac{\Delta_k}{\gamma_k}$.[6] To solve for the pricing equilibrium, we divide every firm n's cost and surplus by γ_n and note that this new market is strategically equivalent to the original one. In this market, every firm's cost is as in B&L, so we apply the algorithm of B&L to the market with $(\tilde{\Delta}_1, \ldots, \tilde{\Delta}_N)$, where $\tilde{\Delta}_n = \frac{\Delta_n}{\gamma_n}$. No other change is needed. B&L's analysis immediately implies the following.

[5] This is an increasing differences condition. When firm n faces price p_m for each worker m, it maximizes $\Delta_n \cdot m - \gamma_n p_m$ or, equivalently, $\frac{\Delta_n \cdot m}{\gamma_n} - p_m$. Therefore, to guarantee that in a competitive equilibrium stronger firms hire better workers (who are more expensive), $\frac{\Delta_n \cdot m}{\gamma_n}$ must have increasing differences. This implies that $\frac{\Delta_n}{\gamma_n}$ is non-decreasing in n.

[6] Similarly, the worst competitive equilibrium from the firms' perspective has $p_n = \sum_{k \leq n} \frac{\Delta_k}{\gamma_k}$.

Claim 1 *When firms' costs are linear but not necessarily identical, B&L's qualitative comparisons regarding workers' utilities and firms' profits between the firm-optimal competitive equilibrium and the pricing equilibrium hold.*

Proof. The firm-optimal competitive equilibrium and the pricing equilibrium are identical to those in B&L's setting with $(\tilde{\Delta}_1, \ldots, \tilde{\Delta}_N)$, so the comparison regarding workers' utilities remains unchanged. The comparison regarding firms' profits also remains unchanged, because each firm n's utility is simply multiplied by γ_n in both equilibria relative to B&L's setting with $(\tilde{\Delta}_1, \ldots, \tilde{\Delta}_N)$. ■

Asymmetric linear costs can, however, cause the difference between agents' utilities in the competitive equilibrium and the pricing equilibrium to be less pronounced than in B&L. This is because the effect of differences in valuations can be mitigated by differences in costs, which is most easily seen by considering the extreme case in which $\frac{\Delta_1}{\gamma_1} = \cdots = \frac{\Delta_N}{\gamma_N}$.

Claim 2 *When $\frac{\Delta_1}{\gamma_1} = \cdots = \frac{\Delta_N}{\gamma_N}$, each firm's profit and workers' average (aggregate) utilities in the pricing equilibrium equal those in the firm-optimal competitive equilibrium.*

Proof. Let $\alpha = \frac{\Delta_1}{\gamma_1} = \cdots = \frac{\Delta_N}{\gamma_N}$. Every worker n's utility in the firm-optimal competitive equilibrium is $\alpha(n-1)$, so firm n's profit is $n\Delta_n - \gamma_n \alpha(n-1) = \Delta_n$. To obtain the pricing equilibrium, consider B&L's algorithm. Because $\frac{\Delta_1}{\gamma_1} = \cdots = \frac{\Delta_N}{\gamma_N}$, firms have identical densities and exhaust their probabilities simultaneously. Therefore, there is only one interval of competition. On this interval, each firm's density is $\frac{1}{\alpha(N-1)}$, so firms compete by mixing uniformly on $[0, \alpha(N-1)]$. Because there is no atom at 0 (no firm chooses 0 with positive probability), each firm obtains the worst worker by offering 0, so every firm n's payoff is Δ_n, just like in the firm-optimal competitive equilibrium. The average of workers' utilities is the middle of the interval, $\frac{\alpha(N-1)}{2}$. This is equal to the average of workers' utilities in the firm-optimal competitive equilibrium, $\frac{\sum_{n=1}^{N} \alpha(n-1)}{N}$. ■

In this extreme example it is easy to see why workers' utilities in the pricing equilibrium are compressed relative to their utilities in the firm-optimal competitive equilibrium: in the firm-optimal competitive equilibrium workers' utilities are spaced evenly on the interval $[0, \alpha(N-1)]$, whereas in the pricing equilibrium workers' expected utilities are spaced evenly on the interval $\left[\frac{\alpha(N-1)}{N+1}, \frac{\alpha(N-1)N}{N+1}\right]$, whose length is $\frac{\alpha(N-1)^2}{N+1} < \frac{\alpha(N+1)(N-1)}{N+1} = \alpha(N-1)$.

When $\tilde{\Delta}_1, \ldots, \tilde{\Delta}_n$ are close to each other, asymmetric linear costs can also make competition less localized than in the setting of B&L. As the proof of Claim 2 shows, even when $\Delta_1, \ldots, \Delta_n$ differ, so that with symmetric linear costs competition is localized, if $\tilde{\Delta}_1 = \cdots = \tilde{\Delta}_n$ then *all* firms compete on the same interval.

4 Firms with Symmetric Non-Linear Costs

A key feature of B&L's pricing equilibrium is that workers' average utilities decrease in the pricing equilibrium relative to the firm-optimal competitive equilibrium. With non-linear costs this is not always the case, and even with identical non-linear costs average utilities may increase. This is shown in Examples 1 and 2 below.

Because B&L's algorithm requires firms' marginal costs to be constant, non-linear costs mean that the algorithm cannot be applied directly to solve for a pricing equilibrium. Nevertheless, a change-of-variable argument can be used to obtain from B&L's pricing and competitive equilibria the corresponding equilibria of markets in which firms have identical non-linear costs. The following proposition formalizes the change-of-variable argument, and the corollary that follows it applies the argument to study the equilibria of markets in which firms have identical non-linear costs.[7]

Proposition 1 *Let $f\colon [0,\infty) \to [0,\infty)$ be a strictly increasing bijection. Let $\mathbf{G} = (G_1, \ldots, G_N)$ and $\hat{\mathbf{G}} = (\hat{G}_1, \ldots, \hat{G}_N)$ be two strategy profiles such that for every $n \le N$ and $x \in [0,\infty)$, $\hat{G}_n(x) = G_n(f(x))$. Let $p_1, \ldots, p_N$ and $\hat{p}_1, \ldots, \hat{p}_N$ be two profiles of prices such that for every $n \le N$, $\hat{p}_n = f^{-1}(p_n)$.*

1. *$\mathbf{G}$ is a pricing equilibrium of the "linear market," in which every firm's cost equals its score, if and only if $\hat{\mathbf{G}}$ is a pricing equilibrium of the "f-market," in which every firm's cost is f. Moreover, firms have the same payoffs in both equilibria.*

2. *$p_1, \ldots, p_N$ are competitive equilibrium prices of the linear market if and only if $\hat{p}_1, \ldots \hat{p}_N$ are competitive equilibrium prices of the f-market. Moreover, firms have the same payoffs in both equilibria.*

Proof. Denote by $u_n(x)$ firm n's expected payoff in the linear market when it chooses score x and all other firms choose scores according to $\mathbf{G}$. Similarly, denote by $\hat{u}_n(x)$ firm n's expected payoff in the f-market when the firm chooses score x and all other firms choose scores according to $\hat{\mathbf{G}}$. Then, by definition of $\hat{\mathbf{G}}$, for any score $x \ge 0$ at which no firm has an atom we have

$$\hat{u}_n(x) = \Big(1 + \sum_{k \ne n} \hat{G}_k(x)\Big)\Delta_n - f(x) = \Big(1 + \sum_{k \ne n} G_k(f(x))\Big)\Delta_n - f(x) = u_n(f(x)).$$

[7] A similar argument can also be used to obtain the pricing and competitive equilibria when firms' non-linear costs differ by a multiplicative constant.

Hence, a score $x \geq 0$ is a profitable deviation from $\hat{G}_n$ for firm n in the f-market when all other firms choose scores according to $\hat{\mathbf{G}}$ if and only if $f(x)$ is a profitable deviation from G_n for firm n in the linear market when all other firms choose scores according to $\mathbf{G}$.[8] Regarding payoffs, the preceding equality also means that

$$\sup_{x \geq 0} \hat{u}_n(x) = \sup_{x \geq 0} u_n(x),$$

because f is a bijection, which implies the coincidence of firms' payoffs in the two equilibria. This shows the first part of the proposition. For the second part, suppose that $p_1, \ldots, p_N$ are competitive equilibrium prices in the linear market, and denote by $\tilde{n}$ the worker allocated to firm n in this competitive equilibrium. Then, $\hat{p}_1, \ldots, \hat{p}_N$ along with the same allocation of workers to firms form a competitive equilibrium in the f-market. Indeed, suppose to the contrary that in the f-market with prices $\hat{p}_1, \ldots \hat{p}_N$ there is a firm $n \leq N$ that strictly prefers worker m to worker $\tilde{n}$, that is, $\Delta_n \cdot m - f(\hat{p}_m) > \Delta_n \cdot \tilde{n} - f(\hat{p}_n)$. Then

$$\Delta_n \cdot m - p_m = \Delta_n \cdot m - f(\hat{p}_m) > \Delta_n \cdot \tilde{n} - f(\hat{p}_n) = \Delta_n \cdot \tilde{n} - p_n,$$

so firm n strictly prefers worker m to worker $\tilde{n}$ in the linear with prices $p_1, \ldots, p_N$, a contradiction. ■

Corollary 1 *In the f-market,*

1. *There is a unique pricing equilibrium.*

2. *The pricing equilibrium can be constructed in two steps. First, construct B&L's pricing equilibrium in the linear market. Second, transform the pricing equilibrium from the first step as described in Proposition 1.*

3. *In the pricing equilibrium, every firm chooses scores from an interval.*

4. *The firm-optimal competitive equilibrium is $\hat{p}_1, \ldots, \hat{p}_N$, where $\hat{p}_n = f^{-1}\left(\sum_{k<n} \Delta_k\right)$ for every $n \leq N$.*

Proof. Part 1 follows from Proposition 1 and the uniqueness of the pricing equilibrium in the linear market. Part 2 follows directly from Proposition 1. Part 3 follows from Proposition 1 and the fact that in the pricing equilibrium of the linear market, every firm

[8] If x is a profitable deviation, then so are all scores slightly higher than x, by continuity of firms' costs. Therefore, if a profitable deviation exists we can find a score that is a profitable deviation and at which no firm has an atom.

chooses scores from an interval. For part 4, recall that, as discussed in Section 3, the firm-optimal competitive equilibrium in the linear market is $p_1, \ldots, p_N$, where $p_n = \sum_{k<n} \Delta_k$ for every $n \leq N$. Now apply Proposition 1. ■

Proposition 1 and its corollary allow us to compare workers' average expected utility in the pricing equilibrium to their average utility in the firm-optimal competitive equilibrium. The following result shows that in many markets workers can be made better off on average in the pricing equilibrium by making firms' costs sufficiently convex.

Claim 3 *Suppose that $G_1(0) < 1$ in the pricing equilibrium $G = (G_1, \ldots, G_N)$ of the linear market. Then, in the market in which every firm's cost of x is x^a (the x^a-market), workers' average expected utility is higher in the pricing equilibrium than in the firm-optimal competitive equilibrium, for any sufficiently large a. As a tends to infinity, the difference between workers' average expected utility in the pricing equilibrium and the firm-optimal competitive equilibrium converges to $\frac{1-G_1(0)}{N}$.*

Proof. Let $p_1, \ldots, p_N$ be the firm-optimal competitive equilibrium prices in the linear market. In particular, $p_1 = 0$. By Proposition 1, $\hat{p}_1, \ldots, \hat{p}_N$, where $\hat{p}_n^a = p_n^{1/a}$ for every $n \leq N$ and $a > 0$, are the firm-optimal competitive equilibrium prices in the x^a-market. Because $\hat{p}_1^a = 0$ and $\lim_{a\to\infty} p_n^{1/a} = 1$ for any $n > 1$, for any $\varepsilon > 0$ we have $\sum_{n=1}^{N} \hat{p}_n^a < n-1+\varepsilon$ for all sufficiently large a. By Proposition 1, the strategy profile $\hat{G}^a = (\hat{G}_1^a, \ldots, \hat{G}_N^a)$, where $\hat{G}_n^a(x) = G_n(x^a)$ for every $n \leq N$ and $x \in [0, \infty)$, is the pricing equilibrium of the x^a-market. For every $n \leq N$ and $\delta > 0$,

$$\lim_{a\to\infty} \hat{G}_n^a(1+\delta) = \lim_{a\to\infty} G_n((1+\delta)^a) = G_n(\infty) = 1, \text{ and}$$
$$\lim_{a\to\infty} \hat{G}_n^a(1-\delta) = \lim_{a\to\infty} G_n((1-\delta)^a) = G_n(0).$$

Because $G_n(0) = 0$ for any $n > 1$, we have that for any $\varepsilon > 0$ the sum of the expected scores according to $\hat{G}^a$ is higher than $(n-1)+1-G_1(0)-\varepsilon$, for sufficiently large a. For sufficiently small $\varepsilon > 0$ and sufficiently large a, we have that $n - 1 + \varepsilon < (n-1) + 1 - G_1(0) - \varepsilon$. In addition, because for any $\delta > 0$ no firm chooses scores above $1+\delta$ for sufficiently large a, it is straightforward to see that as a tends to infinity, the difference in the average expected utilities converges to $\frac{1-G_1(0)}{N}$. ■

The condition $G_1(0) < 1$ in Proposition 3 means that firm 1 is "active" in the pricing equilibrium, that is, it chooses positive bids with positive probability. The condition is satisfied when $\Delta_2 \neq \Delta_3$ or $\Delta_2 - \Delta_1$ is sufficiently small.[9] What underlies Proposition 3 is

[9]When $\Delta_2 \neq \Delta_3$ firms 2 and 3 exhaust their probabilities at different scores, so firm 2 competes against firm 1 on an interval of positive length. When $\Delta_2 - \Delta_1$ is sufficiently small, firm 1 finds it profitable to compete whenever firm 2 does, and therefore chooses scores from an interval of positive length.

the fact that firms compete more aggressively at scores at which the cost function increases more quickly. This behavior maintains each firm's indifference among the different scores in the support of its equilibrium strategy. When the parameter a increases, x^a becomes more convex, so the competition focuses on higher scores.

The following example demonstrates the content of Proposition 3 in a market with three firms.

Example 1 *There are three workers and three firms, with* $\Delta_1 = 1$, $\Delta_2 = 2$, *and* $\Delta_3 = 3$. *For any* $a \geq 11$, *in the* x^a*-market workers' average expected utility in the pricing equilibrium is higher than in the firm-optimal competitive equilibrium. For* $a = 11$, *worker's utilities (scores) in the firm-optimal competitive equilibrium are* $0, 1,$ *and* $3^{1/11} = 1.11$, *with an average of* 0.702. *Workers' expected utilities (scores) in the pricing equilibrium, are* $0.14, 0.94,$ *and* 1.03, *with an average of* 0.703. *As* a *tends to infinity, the difference in workers' average expected utilities converges to* $\left(1 - G_1\left(0\right)\right)/3 = 1/18$.

The next example shows that workers' average expected utility may be higher in the pricing equilibrium even when firms' costs are not convex.

Example 2 *There are three workers and three firms, with* $\Delta_1 = 4, \Delta_2 = 5,$ *and* $\Delta_3 = 20$. *The firms' common cost function is depicted on the left-hand side of Figure 1. Worker's utilities (scores) in the firm-optimal competitive equilibrium are* $0, 4.9,$ *and* 5.87, *with an average of* 3.59. *Workers' expected utilities (scores) in the pricing equilibrium, which is depicted on the right-hand side of Figure 1, are* $2.26, 4.48,$ *and* 5.09, *with an average of* 3.94.[10]

[10]The common cost function is $c(x) = \frac{5\tan^{-1}(3(x-5))}{\tan^{-1}(15)} + 5$. Firms' equilibrium densities are

$$g_1(x) = \begin{cases} \frac{c'(x)}{\Delta_2} & x_1 > x \\ 0 & \text{otherwise} \end{cases}, \quad g_2(x) = \begin{cases} \frac{c'(x)}{\Delta_1} & x_1 > x \\ \frac{c'(x)}{\Delta_3} & x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases}, \quad \text{and } g_3(x) = \begin{cases} \frac{c'(x)}{\Delta_2} & x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases},$$

where $x_1 = 5 - \frac{1}{3}\tan\left(\frac{2\tan^{-1}(15)}{5}\right)$ and $x_2 = 5 + \frac{1}{3}\tan\left(\frac{2\tan^{-1}(15)}{5}\right)$.

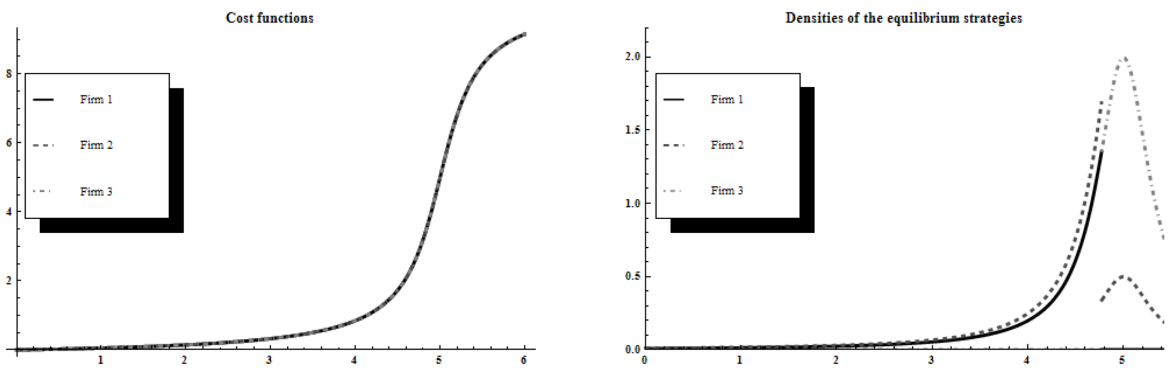

Figure 1: The common cost function (left) and the densities of firms' strategies in the pricing equilibrium (right) of Example 2.

The reason that workers' average utilities increase in the pricing equilibrium relative to the firm-optimal competitive equilibrium is that firms' high marginal costs at high scores result in the two strongest firms concentrating their competition on high scores, which pushes up workers' utilities.

5 Firms with Asymmetric Non-Linear Costs

With asymmetric, non-linear costs, B&L's algorithm cannot be applied to solve for an equilibrium. This is because firms' marginal costs are not constant, and there is no simple transformation that changes the market to one for which B&L's algorithm applies. An alternative approach is to apply a version of the algorithm developed by Siegel (2009). This algorithm can be used to solve for a pricing equilibrium by proceeding from score 0 upward, as long as firms' equilibrium payoffs are known. At each score, the set of actively competing firms is identified, and the equilibrium indifference condition implied by firms' payoffs is used to construct their equilibrium strategies. The complication is that firms need not be active on an interval, and Siegel's (2009) algorithm shows how to overcome this difficulty. Although we have been unable to find a general method of identifying firms' payoffs in a pricing equilibrium, we were able to identify payoffs in some examples.[11]

Example 3 below shows that with asymmetric non-linear costs the utility of a worker who is not the worst worker may increase in the pricing equilibrium relative to the firm-optimal competitive equilibrium, and workers' average utilities may increase as well. The example also shows that some firms may no longer choose scores from an interval. In the example, firm 3 employs a non-interval strategy because its marginal cost for intermediate scores is much higher than its marginal cost for low and high scores. That firms may employ non-interval strategies suggests that asymmetric non-linear costs may lead to non-localized competition.

Example 3 *There are three workers and three firms, with* $\Delta_1 = 4, \Delta_2 = 5,$ *and* $\Delta_3 = 20$*. Firms' cost functions are depicted on the left-hand side of Figure 2. Worker's utilities (scores) in the firm-optimal competitive equilibrium are* $0, 20,$ *and* 40*, with an average of* 20*. Workers' expected utilities (scores) in the pricing equilibrium, which is depicted on the right-hand side of Figure 2, are* $11.06, 23.25,$ *and* 30.77*, with an average of* 21.7*. Both worker 1 and 2's utilities and workers' average utility increase in the pricing equilibrium relative to the firm-optimal competitive equilibrium. In addition, the support of firm* 3*'s strategy is not an interval.*[12]

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[11] We did this by guessing firms' payoffs, running the algorithm of Siegel (2009) assuming these payoffs, and verifying that the resulting strategies form an equilibrium.

[12] Firms' cost functions are $c_1(x) = \frac{x}{5}$, $c_2(x) = \frac{x}{4}$, and $c_3(x) = 20 - \frac{20\tan^{-1}(20-x)}{\tan^{-1}(20)}$. Firms' equilibrium densities are

$$g_1(x) = g_2(x) = \begin{cases} \frac{c_3'(x)}{2\Delta_3} & 0 \le x < x_1 \text{ or } x_2 < x \le x_3 \\ \frac{1}{\Delta_3} & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}, \quad \text{and } g_3(x) = \begin{cases} \frac{1}{\Delta_3} - \frac{c_3'(x)}{2\Delta_3} & 0 \le x < x_1 \text{ or } x_2 < x \le x_3 \\ 0 & \text{otherwise} \end{cases},$$

where $x_1 = 20 - \sqrt{-1 + \frac{10}{\tan^{-1}(20)}}$, $x_2 = 35.2305$, and $x_3 = 40$.

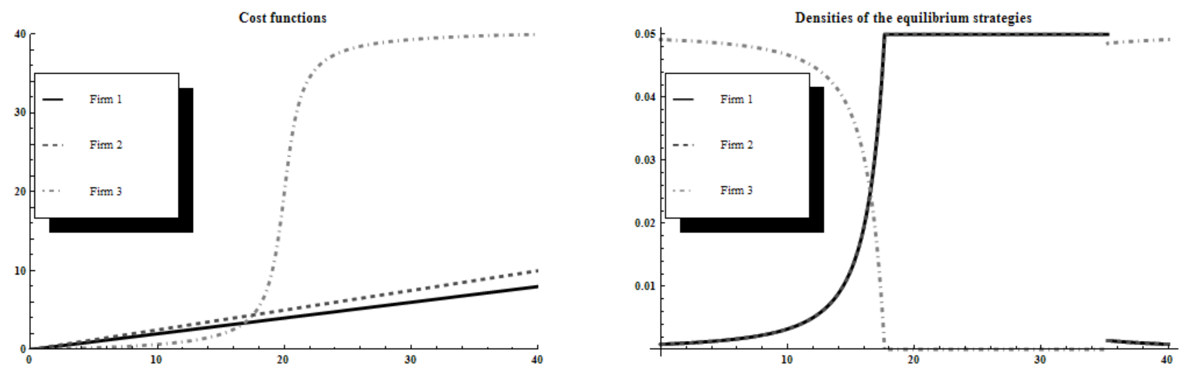

Figure 2: Firms' cost functions (left) and the densities of firms' strategies in the pricing equilibrium (right) of Example 3.

6 Conclusion

Our analysis shows that B&L's simplifying assumption that hospitals are ranked according to expenditures is not without loss of generality. Because non-linearities and asymmetries are common in the real world, our findings suggest that B&L's conclusions should be examined with care. From a modeling perspective, the combination of heterogeneous prizes and asymmetric, non-linear costs presents considerable technical difficulties, and further work is necessary to better understand such models.

References

[1] **Artemov, Georgy**. 2008. "Matching and price competition: would personalized prices help?" *International Journal of Game Theory*, 36(3-4), 321–331.

[2] **Barut, Yasar and Kovenock, Dan.** 1998. "The Symmetric Multiple Prize All-Pay Auction with Complete Information." *European Journal of Political Economy*, 14(4), 627-644.

[3] **González-Díaz, Julio.** 2011. "First-Price Winner-Takes-All Contests." *Optimization*, forthcoming.

[4] **Kojima, Fujito.** 2007. "Matching and Price Competition: Comment." *American Economic Review*, 97(3), 1027-1031.

[5] **Nierdele, Muriel.** 2007. "First-Price Winner-Takes-All Contests." *American Economic Review*, 97(5), 1957-1969.

[6] **Siegel, Ron.** 2009. "All-Pay Contests." *Econometrica*, 77(1): 71-92.

[7] **Siegel, Ron.** 2010. "Asymmetric Contests with Conditional Investments." *American Economic Review*, 100(5), 2230-2260.

[8] **Siegel, Ron.** 2011. "Head Starts in Contests." mimeo.

[9] **Xiao, Jun.** 2011. "Asymmetric All-Pay Contests with Heterogeneous Prizes." mimeo.