Encarnación Algaba, Vito Fragnelli, Joaquín Sánchez-Soriano

Handbook of the Shapley Value

Contents

Chapter 17

The Shapley rule for loss allocation in energy transmission networks

Gustavo Bergantiños

Department of Statistics and Operations Research, University of Vigo

Julio González-Díaz

Department of Statistics, Mathematical Analysis and Optimization and Research group MODESTYA, University of Santiago de Compostela and ITMATI

Ángel M. González-Rueda

Department of Statistics, Mathematical Analysis and Optimization and Research group MODESTYA, University of Santiago de Compostela

Published as a chapter in the Handbook of the Shapley Value, by CRC Press (Taylor & Francis Group), Chapter 17, 369-391 (2019).

Published version available at https://www.crcpress.com/Handbook-of-the-Shapley-Value/Algaba-Fragnelli-Sanchez-Soriano/p/book/9780815374688

Abstract

We consider the problem of loss allocation in energy transmission networks. We introduce the Shapley rule defined as the Shapley value of an associated cooperative game. We study the properties satisfied by the Shapley rule. We compare this rule, in terms of the principles mentioned in the *EU* regulations, with the rules studied in [1]. Finally we apply this rule to the Spanish gas transmission network and carry out a simulation analysis to explore new connections between the different allocation rules.

JEL classification. C7, L95, R48

Keywords. Gas transmission networks, loss allocation, cost allocation, management

---

17.1 Introduction

The analysis and modeling of different aspects of energy transmission networks is a prevalent topic in papers across a wide variety of disciplines. In particular, one important aspect is the study of energy losses in these networks. This issue was recently tackled in [1], where the authors say:

> "A common problem is that, in virtually any network, there are losses whose sources are normally difficult to identify. Thus, one must anticipate them so that they do not lead to deficit in the system. In many cases the transmission network is owned by different agents and, typically, the authorities that manage the network decide how much energy each agent is allowed to lose. This decision should follow some general principles, which would then appear in the relevant regulations. For instance, one would like that the loss allocated to each agent takes into account characteristics of the agents, such as the size of its subnetwork or the amount of energy managed."

Although the analysis of this paper could be applied to any energy transmission network, we develop it using a gas transmission network because our leading example is the Spanish gas transmission network. It is worth noting that the use of natural gas as a source of energy has been rapidly increasing over the past few years. According to a review by British Petroleum in 2013 ([6]), the consumption of natural gas world wide was around the 23.9% of global primary energy consumption. A more recent report published by Enerdata in 2017 (see [8]) also reports a share over 20% of natural gas.

Going back to the issue of energy losses and, more specifically, energy losses in gas networks, [1] go on to say:

> "Different networks have different estimates on the percentage of gas/electricity that is lost during transportation. In Spain, for instance, this estimate is 0.2% for the gas transported in the high pressure gas network and similar figures have been reported in other countries. In order to prevent the ensuing monetary losses, a standard approach in energy networks is to withhold at the entry points a pre-set percentage of the gas/electricity entering the network; by doing this, the energy companies that use the network for transportation are the ones effectively assuming the associated cost in the first instance. In particular, in the Spanish high pressure gas network the pre-set percentage withhold to anticipate the estimated losses is precisely 0.2%. In monetary terms, the annual cost of the gas entering the Spanish gas network is around 12000 millions of Euro, which results in approximately 25 millions of Euro in losses in the transmission network.
> It is precisely at this point where the main question we try to address in this paper arises. Since a gas network is typically owned by different agents, called haulers, it must be decided how to share the withhold gas among them. More precisely, it must be decided, for each agent, the percentage of the gas entering his subnetwork that he can lose. Note that it is not possible to let each agent lose the same percentage that has been withhold for the entire network. Since most gas entering the network crosses several subnetworks, this naive approach would result in allowing the agents to lose, in aggregate, more gas than the withhold amount."

The Spanish regulation presents an incentive mechanism to induce haulers to reduce the losses (see [5, page 106656]). On a yearly basis the following values are computed: A_h is the 'allowed' loss assigned to each hauler h; L_h is the real loss of each hauler h (it is computed as the balance between entries and exits of gas in his subnetwork); given a price p per unit of gas, the haulers pay $p\,(L_h - A_h)$ when $L_h - A_h > 0$ and receive $\frac{p}{2}\,(A_h - L_h)$ when $L_h - A_h \leq 0$. Therefore, the definition of the rule to assign the 'allowed' losses is a relevant issue for the management of gas transmission networks.

Regulation (EC)(no. 55/2003, [13]), from the European Union mentions some principles that should be followed by the national and international regulations regarding the natural gas market. The analysis in [1] starts with the definition of four different allocation rules for energy losses, which are then compared conducting a thorough axiomatic analysis that builds upon the above principles. Besides, an application using data from the Spanish gas transmission network is presented, comparing the allocation proposed by the different rules. The main conclusion of that paper is that the rule that behaves worst (in terms of the EU principles) is the so called *aggregate edge's rule*. This rule was replaced in Spain by the *flow's rule* because of the strong opposition of the small haulers (on the grounds that it favored big haulers). The *proportional tracing rule* and the *edge's rule* behave better than the flow's rule (in terms of the EU principles), with the former seeming slightly preferable.

In this paper we present a new rule, the Shapley rule, obtained as the Shapley value of a cooperative game with transferable utility that can be associated to each gas loss problem. Then, we closely follow the analysis in [1]. We first study the axiomatic behavior of the Shapley rule with respect to the same set of axioms, finding that this new rule is not as good as those performing best in the original paper: the proportional tracing rule and the edge's rule. Second, we find that, in the application from the Spanish network, the allocation proposed by the Shapley rule is very similar to that proposed by the proportional tracing rule. Motivated by this similarity, we build upon the real data from the Spanish network to conduct a simulation analysis over 10000 randomly generated modifications of it. The analysis of the resulting loss allocations shows that the average correlation between the allocation proposed by the Shapley rule the one proposed by the proportional tracing rule is over 0.99, while the minimum correlation between these two rules found in those 10000 simulations is still over 0.9. This reinforces the idea that there must be some common mechanism underlying both rules, which should definitely be explored more deeply. This is specially so if we take into account that the second highest average correlation, although still very high, is at 0.97, whereas the second highest minimum correlation for any other pair of tariffs across the 100000 simulations is just over 0.6.

The use of the Shapley value in this kind of settings is not new. It has already been used in many allocation problems. The basic idea is always the same. One starts associating to each problem a cooperative game with transferable utility. Then, the Shapley rule for the given problem is defined as the Shapley value of the associated cooperative game. This approach has been followed, for instance, in airport problems (see [11]), queuing problems (see [12] and [7]), and minimum cost spanning tree problems (see [10] and [2]). The current paper contributes to this strand of literature by defining, and studying, the Shapley rule for energy transmission networks.

In the associated cooperative game with an energy transmission network the agents are the haulers. The value of a coalition T of haulers should be defined as the loss that haulers in T can have by "themselves". Several definitions are possible. We give a definition inspired in the approach taken in

[9] for flow games. In their model there is also a set of agents who own the different edges of the network and the value of a group of agents T is defined as the maximum amount of flow that can be transported (from the source to the sink) using only edges belonging to agents in T. We apply the same principle to our model. We define the value of coalition T as loss associated with the maximum demand that can be satisfied using only edges of haulers in T, *i.e.*, the loss associated with the maximum amount of gas that can be transported from suppliers to consumers without exceeding the capacities and demands of suppliers and consumers, respectively.

The paper is structured as follows. In Section 17.2 we summarize the relevant characteristics of the management and operation of a gas transmission network and the formal mathematical model. In Section 17.3 we introduce the Shapley rule. In Section 17.4 we present different properties, motivated by some principles stated in EU regulations. In Section 17.5 we discuss the behavior of the Shapley rule with respect to these properties and principles. In Section 17.6 we present the application to the Spanish gas transmission network.

17.2 The model

In this section we introduce the mathematical model associated with a loss energy problem. In order to make this paper self-contained, we formally introduce all the elements of the model, but we do so in a very concise way. Also, to facilitate the comparison with the analysis in [1], we closely follow the notations and formal definitions in that paper. We refer the reader to sections 2 and 3 of [1] for a more detailed explanation of all concepts introduced below.

Since our motivating example comes from the Spanish gas transmission network, the exposition is carried out for gas networks. Yet, our analysis and results may be applied to other energy transmission networks. As far as this paper is concerned, a gas network may be seen as a graph, composed of nodes and edges. There are three types of nodes: demand nodes, in which some gas leaves the network; supply nodes, in which some gas enters the network; and the rest of the nodes, in which the gas that enters and leaves coincide. Edges represent pipes. Each pipe belongs to a hauler and a hauler may own several pipes.

In order to develop our analysis we assume that, for each pipe, its volume and the amount of gas flowing through it are known. The flow represents the total amount of energy each pipe carries during a given period of time (which we measure in GWh/d). The Technical System Manager decides how the gas flows through the network. The first step is to obtain the demands at the different nodes. Then, following some criteria, the Technical System Manager decides the gas that should be introduced at each supply node and how the gas should be routed so that the total demand is fulfilled. The volume of a pipe just depends on its length and its diameter. It is worth noting that the total amount of gas that can flow through a pipe is not just a function of its volume. Since natural gas is a compressible fluid, the capacity of a pipe crucially depends on the construction materials and the maximum pressure they can support.

A flow configuration, based on some realistic scenario of demands, is an important part of the input to a loss allocation rule. In energy networks, it is usual to work with reference scenarios with high/peak demand. This is the case of the data of the Spanish gas network analyzed in Section 17.6. The way to choose the reference scenario, although crucial to obtain cost-reflective loss allocations, is not important for the theoretical analysis of this paper. Once a methodology is chosen to allocate the losses, it can be applied to individual scenarios and also to compute averages over sets of reference scenarios to get more representative allocations.

Given a gas network configuration, we can estimate the total loss of the system, say L, during a given year. This total loss L has to be assigned to the haulers. Let A_h be the loss assigned to hauler h. Let L_h be the real loss measured in the subnetwork of hauler h during this year. In the Spanish network, given a price p per unit of gas, the haulers pay $p(L_h - A_h)$ when $L_h - A_h > 0$ and receive $\frac{p}{2}(A_h - L_h)$ when $L_h - A_h \leq 0$.

17.2.1 The mathematical model

Let $U = \{1, 2, 3, \ldots\}$ be the (infinite) set of possible *nodes*. A *graph* is a pair $g = (N, E)$ where $N \subset U$ is the (finite) set of nodes and E is a set of edges, defined as ordered pairs in N, *i.e.*, $E \subset \{(i, j) : (i, j) \in N \times N$ and $i \neq j\}$. More generally, a multigraph is also a pair $g = (N, E)$, but where the set of edges is a multiset $E \subset N \times N \times \mathbb{N}$. In particular, we say that two edges (i, j, n) and (i', j', n') are part of a multiedge if $i = i'$, $j = j'$, and $n \neq n'$. We say that E does not have multiedges if the projection of E on $N \times N$ is injective.

A *path* in g between i and j is a sequence of $l > 1$ nodes $\{k_1, \ldots, k_l\}$ such that $i = k_1$, $j = k_l$, and $(k_{q-1}, k_q) \in E$ for all $q \in \{2, \ldots, l\}$. A *simple path* in g between i and j is a path where all nodes are different. For the sake of notation we often identify a path with the set of edges $\{(k_{q-1}, k_q)\}_{q \in \{2, \ldots, l\}}$. A graph g is *connected* if for each pair of nodes i and j there is a path between i and j in the undirected version of g. We omit the trivial extension of these definitions for multigraphs.

A *gas loss problem* G is a 5-tuple $(g, v, f, \mathcal{H}, \alpha)$ consisting in the following elements:

1. The multigraph $g = (N, E)$ represents the *gas network*.

   We assume that g is a directed and connected graph without cycles, where the directions of the edges are determined by the gas flows in the given scenario. If $e = (i, j, l) \in E$, then there may be gas flowing from i to j.

2. $v = (v_e)_{e \in E}$ where, for each $e \in E$, $v_e > 0$ denotes the *volume* of e.

3. $f = (f_e)_{e \in E}$ is the *flow configuration* where, for each $e \in E$, $f_e \geq 0$ denotes the flow of gas through e. We assume that $\sum_{e \in E} f_e > 0$.

4. $\mathcal{H} = (H, \{E_h\}_{h \in H})$ is the *hauler structure*, where H denotes the set of haulers and, for each $h \in H$, E_h denotes the (possibly empty) set of edges of hauler h. In particular, $E = \bigsqcup_{h \in H} E_h$.

5. $\alpha \in [0, 1]$ denotes the *proportion of gas* allowed to be lost by the set of haulers.

For the sake of notation, graphs are used for most of the exposition, with multigraphs being used only when they make a difference. Further, we assume that the set H is infinite, although in each given problem only a finite number of them will own edges. This is convenient in the study of some properties of allocation rules. Yet, in the examples we just mention those haulers who own some edge in the given problem.

The example below is borrowed from [1]:

Example 1 *Let G be the gas problem where*

1. $g = (N, E)$, *where the set of nodes is* $N = \{s_1, s_2, 1, c_1, c_2\}$ *and the set of edges is* $E = \{(s_1, 1), (1, c_1), (s_2, 1), (1, c_2)\}$.

2. $v_{(s_1,1)} = v_{(s_2,1)} = v_{(1,c_1)} = v_{(1,c_2)} = 100$.

3. $f_{(s_1,1)} = 20$, $f_{(s_2,1)} = 80$, $f_{(1,c_1)} = 60$, *and* $f_{(1,c_2)} = 40$.

4. $\mathcal{H} = (H, \{E_h\}_{h \in H})$, *where* $H = \{h_1, h_2, h_3\}$ *and* $E_{h_1} = \{(s_1, 1), (1, c_1)\}$, $E_{h_2} = \{(s_2, 1)\}$, *and* $E_{h_3} = \{(1, c_2)\}$.

5. $\alpha = 0.1$.

This gas problem is represented in Figure 17.1 and will be used as a running example to illustrate some concepts and definitions. $\Diamond$

We now introduce some terminology. For each $i \in N$, we denote by Q_i the gas balance at node i, *i.e.*, the amount of gas leaving node i minus the amount of gas arriving at node i. Formally,

$$Q_i = \sum_{(i,j) \in E} f_{(i,j)} - \sum_{(j,i) \in E} f_{(j,i)}.$$

The set of *suppliers* $S \subset N$ of the gas problem G is defined as the set of nodes $s \in N$ such that $Q_s > 0$. On the other hand, the set of *consumers*

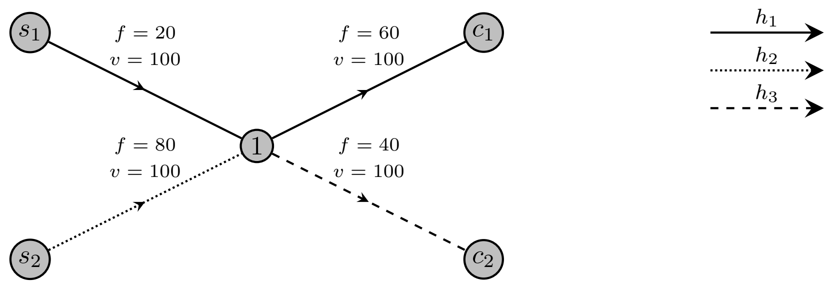


FIGURE 17.1: Representation of the gas problem in Example 1.

$C \subset N$ is defined as the set of nodes $c \in N$ such that $Q_c < 0$. For the rest of nodes $i \in N \setminus (S \cup C)$, we have that $Q_i = 0$. We make the natural assumption that total supply and total demand are balanced, namely,

$$\sum_{s \in S} Q_s = -\sum_{c \in C} Q_c \quad \text{or, equivalently,} \quad \sum_{i \in N} Q_i = 0.$$

The total loss allowed to the haulers is $L = \alpha \sum_{s \in S} Q_s$. The *flow carried* by each hauler $h \in H$, denoted by f_h, is defined as the gas that reaches one of the edges of hauler h from outside, that is, from some provider $s \in S$ or from an edge of another hauler. Formally, we first define, for each node $i \in N$ and each hauler $h \in H$, $Q_i^h = \max\{\sum_{(i,j) \in E_h} f_{(i,j)} - \sum_{(j,i) \in E_h} f_{(j,i)}, 0\}$; if no edge of hauler h contains node i we define $Q_i^h = 0$. Then, for each $h \in H$,

$$f_h = \sum_{i \in N} Q_i^h.$$

In particular, $f_h = 0$ whenever $E_h = \emptyset$.[1]

Given a gas problem G and a pair $(s, c) \in S \times C$, we define $P(s, c)$ as the set of simple paths in g from s to c. We denote by $P(S, C)$ the set of all simple paths from suppliers to consumers. Namely,

$$P(S, C) = \bigcup_{(s,c) \in S \times C} P(s, c).$$

We now want to define an important notion for our analysis that we call *hauler's influence network*, which, given a hauler h, would contain all edges whose gas might either reach some edge in E_h or come from some edge in E_h. Formally, for each $h \in H$, we define $\mathcal{N}^h = (g^h, v^h, f^h)$, as the subnetwork of

[1] There are alternative ways to define the notion of "flow carried by a hauler", but, as far as our analysis is concerned, they would lead to similar results. Our formulation is the one implicit in the Spanish Regulations ([3, 5]).

(g, v, f) where $g^h = (N^h, E^h)$ and

$$\begin{aligned} E^h &= \{e \in E: \text{ there is } p \in P(S,C) \text{ with } e \in p \text{ and } p \cap E_h \neq \emptyset\}, \\ N^h &= \{i \in N : i \in e \text{ for some } e \in E^h\}, \\ v^h &= (v_e)_{e \in E^h}, \\ f^h &= (f_e)_{e \in E^h}. \end{aligned}$$

Sometimes we slightly abuse language and refer to an *edge's influence network*, to mean the influence network that would have a hauler who owned only that edge.

Example 1 *(cont.) Going back to the gas problem in Figure 17.1, we have that* $Q_{s_1} = 20$, $Q_{s_2} = 80$, $Q_1 = 0$, $Q_{c_1} = -60$, *and* $Q_{c_2} = -40$. *Thus,* $S = \{s_1, s_2\}$ *and* $C = \{c_1, c_2\}$. *The table below contains the different* Q_i^h *flow balances and Figure 17.2 represents the influence networks corresponding to this example.*

Q_i^h	s_1	s_2	1	c_1	c_2	f_h
h_1	20	0	40	0	0	60
h_2	0	80	0	0	0	80
h_3	0	0	40	0	0	40

$\diamondsuit$

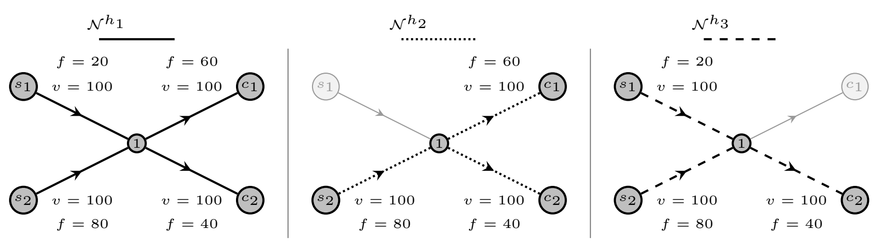


FIGURE 17.2: Illustration of the hauler's influence networks of Example 1.

17.3 The Shapley rule

In [1] the authors study four rules that provide, for each gas loss problem, an allocation of the allowed loss among the different haulers. These rules, whose definitions can be seen in [1], are the following: the flow's rule, R^{flow}, the aggregate edge's rule, R^{Aedge}, the edge's rule, R^{edge}, and the proportional tracing rule, $R^{\Gamma^{pt}}$.

In this section we introduce a new allocation rule: the Shapley rule. In order to do it we first associate, to each gas loss problem a cooperative game with transferable utility, and then study the Shapley value of the associated game.

We start with some preliminaries on cooperative games. A cooperative game with transferable utility, briefly a *TU* game, is a pair (H, l) where H is the set of agents and, for each $T \subset H$, $l(T)$ denotes the amount that agents in T can obtain by themselves. We assume that $l(\emptyset) = 0$.

The Shapley value introduced in [14] is, by far, the most studied allocation rule in cooperative game theory. It associates, to each *TU* game (H, l) a vector $\mathrm{Sh}(H, l) \in \mathbb{R}^H$ such that, for each $h \in H$,

$$\mathrm{Sh}_h(H, l) = \sum_{T \subset H \setminus \{h\}} \frac{|T|!\,(|H| - |T| - 1)!}{|H|!} \left(l(T \cup \{h\}) - l(T)\right).$$

In our context H represents the set of haulers and, for each $T \subset H$, $l(T)$, is the loss that haulers in T can have by "themselves". Although there are several ways in which the $l(T)$ values can be defined, we present a natural one inspired in the approach taken in [9] for flow games. In their model there is also a set of agents who own the different edges of the network and the value of a group of agents T is defined as the maximum amount of flow that can be transported (from the source to the sink) using only edges belonging to agents in T. We apply the same principle to our model. Let $f_G(T)$ denote the maximum demand that can be satisfied using only edges of haulers in T, *i.e.*, the maximum amount of gas that can be transported from suppliers to consumers without exceeding the capacities and demands of suppliers and consumers, respectively. We also assume that the capacity of an edge is bounded by f_e, the total amount of gas flowing through that edge in the gas problem under study. Then, we define $l_G(T) = \alpha f_G(T)$; in particular, $l_G(H) = \alpha f_G(H) = \alpha \sum_{s \in S} Q_s = L$. When no confusion arises we write l instead of l_G.

The Shapley rule, R^{Sh}. For each gas problem G we define *the Shapley rule* as $R^{\mathrm{Sh}}(G) = \mathrm{Sh}(H, l_G)$.

Note that $R^{\mathrm{Sh}}(G) = \alpha \mathrm{Sh}(H, f_G)$.

Consider our running example. We first compute the associated cooperative game l. Hauler 1 can transport by himself 20 units. Since $\alpha = 0.1$, $l_G(1) = 0.1 \cdot 20 = 2$. Haulers 1 and 2 can transport by themselves no more than 60 units. They can do in several ways. For instance, 20 units through the path $\{(s_1, 1), (1, c_1)\}$ and 40 units through the path $\{(s_2, 1), (1, c_1)\}$. Since $\alpha = 0.1$, $l_G(1, 2) = 0.1 \cdot 60 = 6$. Analogously we can obtain that

T	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$l_G(T)$	2	0	0	6	2	4	10

Thus the Shapley rule is $R^{\mathrm{Sh}}(G) = (4, 4, 2)$. In the table below we show, for this example, the Shapley rule and the four rules defined in [1]:

h	f_h	R^{flow}	R^{Aedge}	R^{edge}	$R^{\Gamma^{pt}}$	R^{Sh}
1	60	3.33	5	4	4	4
2	80	4.44	3.33	4	4	4
3	40	2.22	1.66	2	2	2

Although in this example several rules lead to the same allocation, in general the five rules are all different from one another.

17.4 Properties

The main objective of this paper is to study the axiomatic behavior of the Shapley rule, and compare this behavior with that of the other rules studied in [1]. In order to do so, we focus our analysis in precisely the properties introduced in that paper, and refer the reader to the discussions therein for additional insights. Since these properties are inspired in the principles mentioned in different regulations and directives of the European Union regulation, the authors in [1] present the following discussion to provide some additional motivation to the properties and their underlying principles:

In Directive 2003/55/EC of the European parliament and the council of 26 June 2003 ([13]), concerning common rules for the internal market in natural gas, establishes some general principles that must be pursued. Some of them are the following:

1. "tariffs are published **prior** to their entry into force".
2. "**the provision of adequate economic incentives**, using, where appropriate, all existing national and Community tools. These tools may include liability mechanisms to guarantee the necessary investment".
3. "national regulatory authorities should ensure that transmission and distribution tariffs are **non-discriminatory** and **cost-reflective**".
4. "Progressive opening of markets towards **full competition** should as soon as possible remove differences between Member States."

The Spanish regulation ensures that tariffs are published prior to their entry into force. Moreover, since the amount

received or paid by each hauler depends monotonically on their loss (the larger is the loss, the larger is the amount the hauler pays) we can argue that it provides the adequate economic incentives.
Regarding the principles of being non-discriminatory, cost-reflective, and foster competition. We introduce some properties related to these principles."

17.4.1 Cost reflective properties

The first property requires that haulers that do not transport gas do not have any assigned loss and the second one says that if two gas problems only differ on edges without flow, then the losses assigned to each hauler should coincide.

Null hauler (NH). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h \in H$ be such that, for each $e \in E_h$, $f_e = 0$. Then, $R_h(G) = 0$.

Independence of unused edges (IUE). Let the gas problems $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ be such that $H = \bar{H}$ and, for each $h \in H$, $\bar{E}_h = E_h \setminus \hat{E}$, where $\hat{E} \subset E$ satisfies that, for each $e \in E \setminus \hat{E}$, $\bar{f}_e = f_e$ and $\bar{v}_e = v_e$, and, for each $e \in \hat{E}$, $f_e = 0$. Then, $R(G) = R(\bar{G})$.

A cost-reflective rule should not be sensitive to "equivalent" representations of the same network. The next two properties try to capture this idea.

Independence of edge sectioning (IES). Let the gas problems $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ be such that $H = \bar{H}$ and there are $\hat{h} \in H$ and $(i, j) \in E_{\hat{h}}$ satisfying

- $\bar{g} = (\bar{N}, \bar{E})$, where $\bar{N} = N \cup \{l\}$ and $l \notin N$, $\bar{E}_{\hat{h}} = (E_{\hat{h}} \backslash \{(i, j)\}) \cup \{(i, l), (l, j)\}$ and, for each $h \in H \backslash \{\hat{h}\}$, $\bar{E}_h = E_h$, and
- $\bar{f}_{(i,l)} = \bar{f}_{(l,j)} = f_{(i,j)}$, $\bar{v}_{(i,l)} + \bar{v}_{(l,j)} = v_{(i,j)}$, and, for each $e \in E \backslash \{(i, j)\}$, $\bar{f}_e = f_e$ and $\bar{v}_e = v_e$.[2]

Then, for each $h \in H$, $R_h(G) = R_h(\bar{G})$.

Independence of edge multiplication (IEM). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ be such that $H = \bar{H}$ and there are $\hat{h} \in H$, $e = (i, j, m) \in E$, $\bar{e}_1 = (i, j, l_1) \in \bar{E}$, and $\bar{e}_2 = (i, j, l_2) \in \bar{E}$ satisfying

- $\bar{g} = (N, \bar{E})$, where $\bar{E}_{\hat{h}} = (E_{\hat{h}} \setminus \{e\}) \cup \{\bar{e}_1, \bar{e}_2\}$ and, for each $h \in H \backslash \{\hat{h}\}$, $\bar{E}_h = E_h$, and

[2] The condition $\bar{v}_{(i,l)} + \bar{v}_{(l,j)} = v_{(i,j)}$ just reflects that, when a pipe is transversely cut (orthogonally to the direction of the flow), the volume of the resulting two pipes adds up to the volume of the original pipe (and the same flow that was crossing the original pipe is crossing the two pipes in which it has been divided $\bar{f}_{(i,l)} = \bar{f}_{(l,j)} = f_{(i,j)}$).

- $f_e = \bar{f}_{e_1} + \bar{f}_{e_2}$, $v_e = \bar{v}_{e_1} = \bar{v}_{e_2}$, and, for each $e \in E\backslash\{e\}$, $\bar{f}_e = f_e$ and $\bar{v}_e = v_e$.[3]

Then, for each $h \in H$, $R_h(G) = R_h(\bar{G})$.

To prevent haulers from artificially distorting the final allocation of losses, if two haulers engage in some trades affecting their own edges, then the rest of the haulers should not be affected. This implies, in particular, that the loss allocated to a hauler does not depend on who owns the edges different from his own.

Independence by sales (IS). Let $G = (g, v, f, \mathcal{H}, \alpha)$, $\bar{G} = (g, v, f, \bar{\mathcal{H}}, \alpha)$, h_1 and h_2 in H, and $e \in E$ be such that $\bar{E}_{h_1} = E_{h_1}\backslash\{e\}$, $\bar{E}_{h_2} = E_{h_2} \cup \{e\}$, and, for each $h \in H\backslash\{h_1, h_2\}$, $\bar{E}_h = E_h$. Then, for each $h \in H\backslash\{h_1, h_2\}$, $R_h(G) = R_h(\bar{G})$.[4]

Independence of irrelevant changes (IIC). Consider the gas problems $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ and let $h \in H \cap \bar{H}$ be such that $\mathcal{N}^h = \bar{\mathcal{N}}^h$. Then, $R_h(G) = R_h(\bar{G})$.

17.4.2 Non-discriminatory properties

The most standard non-discriminatory principle says that we should offer an equal treatment to equal agents. Some of the following properties deal with formalizations of this general notion.

Symmetry on edges (SE). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, $f_e = f_{\bar{e}}$, and $v_e = v_{\bar{e}}$. Then, $R_h(G) = R_{\bar{h}}(G)$.

Symmetry on paths (SP). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, $v_e = v_{\bar{e}}$, and $\mathcal{N}^h = \mathcal{N}^{\bar{h}}$. Then, $R_h(G) = R_{\bar{h}}(G)$.

The following properties build upon the idea that there should be some kind of proportionality on flow and volume.

Flow proportionality on edges (FPE). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, and $v_e = v_{\bar{e}}$. Then, if $f_{\bar{e}} > 0$, we have

$$R_h(G) = \frac{f_e}{f_{\bar{e}}} R_{\bar{h}}(G).$$

Volume proportionality on edges (VPE). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, and $f_e = f_{\bar{e}}$. Then,

$$R_h(G) = \frac{v_e}{v_{\bar{e}}} R_{\bar{h}}(G).$$

---

[3]In this case, the condition $v_e = \bar{v}_{e_1} = \bar{v}_{e_2}$ just reflects that the original pipe e is being replaced by two pipes identical to it: same volume and same endpoints. The total flow in the network remains unchanged, so these two new pipes, together, carry the same flow as e ($f_e = \bar{f}_{e_1} + \bar{f}_{e_2}$).

[4]The rules satisfying IS have an interesting property, which in [1] is referred to as *edge decomposability*. Namely, these rules can be computed in a two stage procedure. We first decide the allowed loss on each edge and later compute the allowed loss to each hauler adding the amount assigned to each of his edges.

Volume proportionality on paths (VPP). Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, and $\mathcal{N}^h = \mathcal{N}^{\bar{h}}$. Then,

$$R_h(G) = \frac{v_e}{v_{\bar{e}}} R_{\bar{h}}(G).$$

17.4.3 Properties to foster competition

The way in which losses are allocated among haulers should not harm competition among agents. In particular, two haulers should not be better off by merging together.

Merging proofness (MP). Let $G = (g, v, f, \mathcal{H}, \alpha)$, $\bar{G} = (g, v, f, \bar{\mathcal{H}}, \alpha)$, $h_1, h_2 \in H$, and $h \in \bar{H}$ be such that $\bar{E}_h = E_{h_1} \cup E_{h_2}$ and, for each $\hat{h} \in H \setminus \{h_1, h_2\}$, $\bar{E}_{\hat{h}} = E_{\hat{h}}$. Then $R_h(\bar{G}) \leq R_{h_1}(G) + R_{h_2}(G)$.

17.5 Axiomatic behavior of the Shapley rule

We present now the main result of this paper, which shows what properties are satisfied by the the Shapley rule.

Proposition 1 *1. The Shapley rule satisfies NH, IUE, IES, IEM, and SP.*

2. The Shapley rule does not satisfy IS, SE, FPE, VPE, VPP, IIF, IIC, and MP.

Proof 1 *We start by proving statement 1.*

• *NH. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h \in H$ be such that, for each $e \in E_h$ $f_e = 0$. Since the edges of hauler h do not carry flow, they never help to increase the total flow that can be carried between a supplier and a consumer. Thus, for each $T \subset H \backslash \{h\}$, we have that $l_G(T) = l_G(T \cup \{h\})$ and the definition of the Shapley value implies that $R_h^{Sh} = 0$.*

• *IUE. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ be as in the definition of IUE, that is, there is $\hat{E} \subset E$ such that, for each $h \in H$, $\bar{E}_h = E_h \setminus \hat{E}$ and, for each $e \in \hat{E}$, $f_e = 0$.*

Let $T \subset H$ be a set of players. Again, the edges that do not carry flow never help to increase the total flow that can be carried between a supplier and a consumer. Thus, they can be removed for the computation of the TU game associated with $\bar{G}$ and, therefore, for each $T \subset H$, $l_G(T) = l_{\bar{G}}(T)$. Thus, $R^{Sh}(G) = R^{Sh}(\bar{G})$.

• *IES. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \bar{\mathcal{H}}, \alpha)$ be two problems that only differ because there are $\hat{h} \in H$ and $(i, j) \in E_{\hat{h}}$ satisfying that (i, j) is sectioned in two consecutive edges $(i, l), (l, j) \in \bar{E}_{\hat{h}}$.*

Since $f_{(i,j)} = \bar{f}_{(i,l)} = \bar{f}_{(l,j)}$, edge sectioning does not change the maximum flow that can be carried from consumers to suppliers. Then, for each $T \subset H$, $l_G(T) = l_{\bar{G}}(T)$ and, therefore, for each $h \in H$, $R^{Sh}_h(G) = R^{Sh}_h(\bar{G})$.

• *IEM. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (\bar{g}, \bar{v}, \bar{f}, \mathcal{H}, \alpha)$ be two problems that only differ because there are $\hat{h} \in H$ and $e \in E_{\hat{h}}$ satisfying that e is duplicated in two multiedges $e_1, e_2 \in \bar{E}_{\hat{h}}$, with $v_e = \bar{v}_{e_1} = \bar{v}_{e_2}$.*

Since $f_e = \bar{f}_{e_1} + \bar{f}_{e_2}$, edge multiplication does not change the maximum flow that can be carried from consumers to suppliers because we only have to split among $\bar{f}_{e_1}$ and $\bar{f}_{e_2}$ the maximum flow that went through f_e. Then, for each $T \subset H$, $l_G(T) = l_{\bar{G}}(T)$ and, therefore, for each $h \in H$, $R^{Sh}_h(G) = R^{Sh}_h(\bar{G})$.

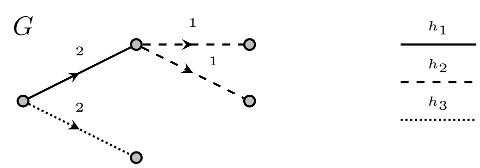
• *SP. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $h, \bar{h} \in H$ be such that $E_h = \{e\}$, $E_{\bar{h}} = \{\bar{e}\}$, $v_e = v_{\bar{e}}$ and $\mathcal{N}^h = \mathcal{N}^{\bar{h}}$.*

Since $\mathcal{N}^h = \mathcal{N}^{\bar{h}}$ we have that $f_e = f_{\bar{e}}$ and, for each $p \in P(S, C)$, $e \in p$ if and only if $\bar{e} \in p$. Then, for each $T \subset H \backslash \{h, \bar{h}\}$ we have $l_G(T \cup h) = l_G(T \cup \bar{h})$. Thus, the definition of the Shapley value implies that $R^{Sh}_h(G) = R^{Sh}_{\bar{h}}(G)$.

Next, we present some counterexamples to prove statement 2.

• *IS. Since IS is stronger than MP (Proposition 1 in [1]) and R^{Sh} does not satisfy MP (see below), R^{Sh} does not satisfy IS.*

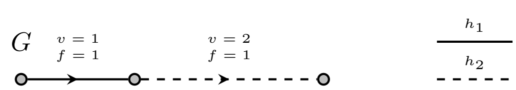
• *SE. Let $G = (g, v, f, \mathcal{H}, \alpha)$ be as in the picture below.*



Problem G is as in the definition of SE, since $h_1 = \{e_1\}$ and $h_3 = \{e_2\}$ with $f_{e_1} = f_{e_2} = 2$ and $v_{e_1} = v_{e_2}$. However, h_3 can satisfy some demand on his own, while h_1 needs h_2. In particular, we get $R^{Sh}_{h_1}(G) = \alpha \neq 2\alpha = R^{Sh}_{h_3}(G)$.

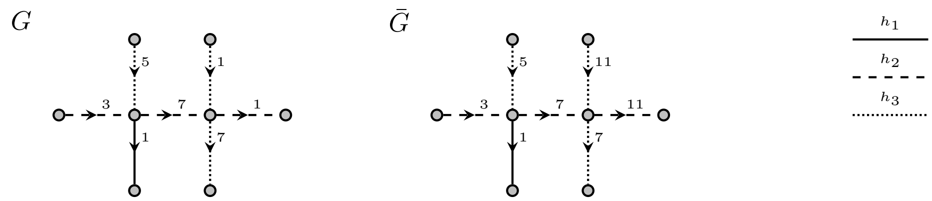
• *FPE and VPE. Since FPE and VPE are stronger than SE (Proposition 1 in [1]) and R^{Sh} does not satisfy SE, R^{Sh} satisfies neither FPE nor VPE.*

• *VPP. Let $G = (g, v, f, \mathcal{H}, \alpha)$, h_1 and h_2 as in the picture below.*



Clearly, $R^{Sh}_{h_2}(G) = R^{Sh}_{h_1}(G) \neq 2R^{Sh}_{h_1}(G) = \frac{v_{h_2}}{v_{h_1}} R^{Sh}_{h_1}(G)$.

• *IIF. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (g, v, \bar{f}, \mathcal{H}, \alpha)$ be as in the picture below.*



Problems G and $\bar{G}$ are as in the definition of IIF. Note that there are two edges where the flow increases and $\mathcal{N}^{h_1} = \bar{\mathcal{N}}^{h_1}$. In this case we get the games

- $l_G(\{h_1\}) = 0$, $l_G(\{h_2\}) = \alpha$, $l_G(\{h_3\}) = \alpha$, $l_G(\{h_1,h_2\}) = 2\alpha$, $l_G(\{h_1,h_3\}) = 2\alpha$, $l_G(\{h_2,h_3\}) = 8\alpha$, $l_G(\{h_1,h_2,h_3\}) = 9\alpha$ *and*

- $l_{\bar{G}}(\{h_1\}) = 0$, $l_{\bar{G}}(\{h_2\}) = 3\alpha$, $l_{\bar{G}}(\{h_3\}) = 7$, $l_{\bar{G}}(\{h_1,h_2\}) = 3\alpha$, $l_{\bar{G}}(\{h_1,h_3\}) = 8\alpha$, $l_{\bar{G}}(\{h_2,h_3\}) = 18\alpha$, $l_{\bar{G}}(\{h_1,h_2,h_3\}) = 19\alpha$.

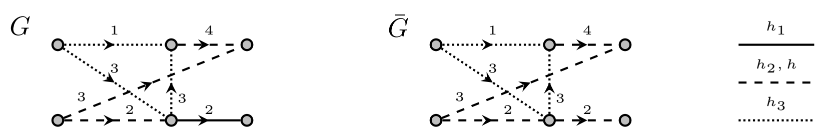
The corresponding Shapley values are so that

$$R^{Sh}_{h_1}(G) = \alpha\frac{4}{6} \neq \alpha\frac{3}{6} = R^{Sh}_{h_1}(\bar{G}).$$

The key is that the marginal contribution of hauler h_1 to hauler h_2 changes from G to $\bar{G}$.

• *IIC. Since IIC is stronger than IIF (Proposition 1 in [1]) and R^{Sh} does not satisfy IIF, R^{Sh} does not satisfy IIC.*

• *MP. Let $G = (g, v, f, \mathcal{H}, \alpha)$ and $\bar{G} = (g, v, f, \bar{\mathcal{H}}, \alpha)$ be as in the picture below.*



Note that $H = \{h_1, h_2, h_3\}$ and $\bar{H} = \{h, h_3\}$ where h is the union of h_1 and h_2. Problems G and $\bar{G}$ are as in the definition of MP. In this case we get the games

- $l_G(\{h_1\}) = 0$, $l_G(\{h_2\}) = 3\alpha$, $l_G(\{h_3\}) = 0$, $l_G(\{h_1,h_2\}) = 5\alpha$, $l_G(\{h_1,h_3\}) = 2\alpha$, $l_G(\{h_2,h_3\}) = 7\alpha$, $l_G(\{h_1,h_2,h_3\}) = 9\alpha$ *and*

- $l_{\bar{G}}(\{h\}) = 5\alpha$, $l_{\bar{G}}(\{h_3\}) = 0$, $l_{\bar{G}}(\{h,h_3\}) = 9\alpha$.

The corresponding Shapley values are so that

$$R^{Sh}_{h}(\bar{G}) = \alpha\frac{42}{6} > \alpha\frac{40}{6} = \alpha\frac{8}{6} + \alpha\frac{32}{6} = R^{Sh}_{h_1}(G) + R^{Sh}_{h_2}(G). \qquad \square$$

EU Principles	Rule / Property	Flow	Aedge	Edge	Prop. Tracing	Shapley
Cost-Reflective	Null hauler	✓	✓	✓	✓	✓
	Ind. Unused Edges	✓		✓	✓	✓
	Ind. Edge Sectioning	✓	✓	✓	✓	✓
	Ind. Edge Mult.	✓		✓	✓	✓
	Ind. Sales			✓	✓	
	Ind. Irr. Changes				✓	
Non-Discriminat.	Symmetry on Edges	✓	✓	✓		
	Symmetry on Paths	✓	✓	✓	✓	✓
	Flow Prop. Edges	✓	✓	✓		
	Volume Prop. Edges		✓	✓		
	Volume Prop. Paths		✓	✓	✓	
	Merging Proofness	✓		✓	✓	
Competition	Merging Proofness	✓		✓	✓	

TABLE 17.1: Behavior of the rules with respect to the different properties.

In Table 17.1 we compare the properties satisfied by the Shapley rule with the properties satisfied by the four rules considered in [1]. The authors then continue discussing, for each of the four rules they study, the "degree of fulfillment" of the three principles. Four degrees were considered: low, normal, high, and very high. We borrow from them the table below, with the addition of one last column for the Shapley rule:

Principle\Rule	Flow	Aedge	Edge	Prop. tracing	Shapley
Cost reflective	Normal	Low	High	Very high	Normal
Non-discriminatory	High	High	Very high	High	Normal
Foster competition	Very high	Low	Very high	Very high	Low

Since the discussion associated to the four rules different from the Shapley value is already included in the analysis in [1], we briefly discuss now the column associated to the Shapley rule. It satisfies the same cost reflective properties as the flow's rule, thus we assign to the Shapley value the same degree in that category. Usually non-discriminatory properties are related with the principle of equal treatment of equals. Then, when comparing symmetry on edges with symmetry on paths, the later takes into account the whole structure of the network, and not just each edge on isolation. Thus, we think that focusing on paths is more reasonable and therefore we assign a normal grade to Shapley rule even though it does not satisfy most of the non-discriminatory properties. Finally, since foster competition has a unique property, the assignment is obvious.

From the table and the above discussion it is clear that the Shapley rule does not exhibit a very good behavior with respect to the different properties and principles, being clearly outperformed by both the proportional tracing rule and the edge's rule.

There are many problems where the Shapley value of an associated cooperative game has many interesting properties compared with other rules in the same setting. We can mention, for instance, airport problems (see [11]), queuing problems (see [12] and [7], and minimum cost spanning tree problems (see [10] and [2]). Nevertheless in our case the Shapley value satisfies less properties than other rules. Of course it could be possible that, if we define the associated cooperative game l_G in a different way, we could obtain a Shapley value with more properties.

In the next section we take a different approach to assess the performance of the Shapley rule, which can be seen as complementary to the one developed in this section. More precisely, we study the allocations the Shapley rule proposes in different problems, a case study with real data and a set of variations of it, and comparing these allocations with the ones proposed by the other four rules.

17.6 Application to the Spanish gas transmission network

17.6.1 Case study with real data

In this section we apply the Shapley rule to the Spanish gas transmission network. We compare the allocation proposed by the Shapley rule with the allocations proposed by the four rules considered in [1]. We build upon the analysis there, and take as benchmark scenario one in which demands follow from reported figures for a hypothetical day of very high demand in the Spanish gas network.[5]

In Figure 17.3 we represent the Spanish gas transmission network. We have boxed the pipes belonging to each hauler, except for hauler $h_{1,}$ who owns all the remaining ones. Hauler h_1 is Enagás, a former public body who initially owned the whole network and still owns more that 90% of the network.

In Tables 17.2, 17.3 and 17.4 we can see the allocations proposed by the Shapley rule and the other rules. We take $\alpha = 0.002$ because is the parameter used in Spain (see [4]).

The three tables contain similar information, but measured in different ways. Moreover, the numbers they contain are the same as in [1], but where an additional column for the Shapley value has been included. Table 17.2 represents the allocated losses measured in gas units, corresponding to the direct application of the different rules to the data of the Spanish scenario under

[5]The computations are derived for the optimal network operation as obtained by the software GANESO^{TM} (developed by researchers at the University of Santiago de Compostela and the Technological Institute for Industrial Mathematics for Reganosa Company). For further details refer to the analysis in [1].

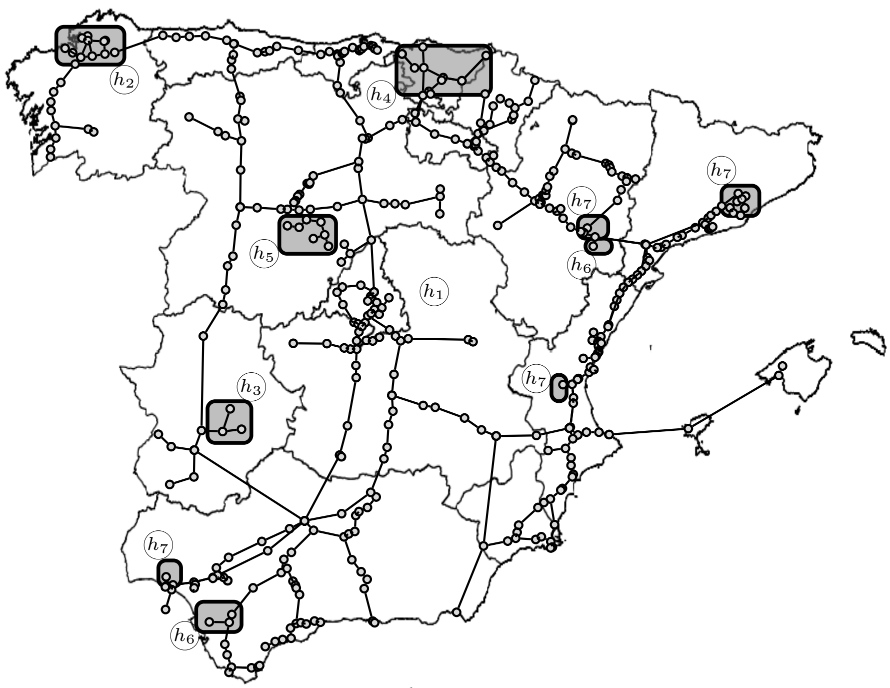


FIGURE 17.3: Haulers of the Spanish gas transmission network.

Gas losses in GWh/d	Network Owned (%)	Flow	Aedge	Edge	Prop. Tracing	Shapley
Enagás (h_1)	91.44	4.55	5.32	5.27	4.72	4.69
Reganosa (h_2)	1.76	0.21	0.0024	0.031	0.21	0.22
Gas Extremadura (h_3)	0.61	0.0071	0.000010	0.00020	0.000073	0.0038
Enagás Transporte del Norte (h_4)	3.54	0.31	0.0086	0.027	0.24	0.27
Transportista Regional Gas (h_5)	1.46	0.016	0.000051	0.0005	0.00052	0.0090
Endesa Gas Transportista (h_6)	0.36	0.0045	0.0000019	0.000029	0.000035	0.0024
Gas Natural (h_7)	0.82	0.24	0.00095	0.0062	0.17	0.14

TABLE 17.2: Gas loss allocated to the haulers (GWh/d) with $\alpha = 0.002$.

Percentage of gas losses (%)	Network Owned (%)	Flow	Aedge	Edge	Prop. Tracing	Shapley
Enagás (h_1)	91.44	85.19	99.77	98.77	88.37	87.88
Reganosa (h_2)	1.76	3.97	0.046	0.59	3.95	4.03
Gas Extremadura (h_3)	0.61	0.13	0.00019	0.0037	0.0014	0.072
Enagás Transporte del Norte (h_4)	3.54	5.74	0.16	0.51	4.44	5.11
Transportista Regional Gas (h_5)	1.46	0.31	0.00096	0.0094	0.0098	0.17
Endesa Gas Transportista (h_6)	0.36	0.083	0.000035	0.00055	0.00066	0.046
Gas Natural (h_7)	0.82	4.58	0.018	0.12	3.23	2.69

TABLE 17.3: Percentage of gas loss allocated to the haulers.

Monetary equivalent in millions of €	Network Owned (%)	Flow	Aedge	Edge	Prop. Tracing	Shapley
Enagás (h_1)	91.44	49.77	58.30	57.71	51.64	51.35
Reganosa (h_2)	1.76	2.32	0.027	0.34	2.31	2.36
Gas Extremadura (h_3)	0.61	0.077	0.00011	0.0022	0.00080	0.042
Enagás Transporte del Norte (h_4)	3.54	3.35	0.095	0.30	2.60	2.99
Transportista Regional Gas (h_5)	1.46	0.18	0.00056	0.0055	0.0057	0.098
Endesa Gas Transportista (h_6)	0.36	0.049	0.000020	0.00032	0.00039	0.027
Gas Natural (h_7)	0.82	2.68	0.010	0.068	1.89	1.57

TABLE 17.4: Annual monetary equivalent, assuming 1 GWh/d = 30000 €.

consideration. Table 17.3 represents the percentage allocated to each hauler. Finally, Table 17.4 represents the estimation of the annual monetary equivalent, provided that the same demands repeat each and every day. Since the scenario under consideration comes from a peak day, whose demand is around twice the demand of an average day, one would get more realistic estimations after dividing by two the amounts in Table 17.4. In practice one might apply the chosen rule on a daily basis and then add up the daily allocations to get the annual loss allocation.

The aggregate edge's rule assign 99.77% of the allocated losses to Enagás, which we believe is unfair. As it was argued in [1] the aggregate edge's rule size discriminates, penalizing small haulers and favoring mergers, which hurts competition. This probably explains why most Spanish haulers strongly opposed to the aggregate edges rule until it was finally replaced by the flow's rule.

In this case we can see that the allocation proposed by the Shapley rule is quite similar to the one proposed by the proportional tracing rule. In the next section we further explore this connection.

17.6.2 Simulation study building upon the real data

Given the results in the analysis above, it is natural to wonder whether or not the similarity between the allocations proposed by the Shapley rule and the proportional tracing rule is just a coincidence for the given data. In order to get additional evidence, we have run a simulation study based on the original scenario, but where relevant data of the problem are randomly modified. More precisely, we have generated 10000 scenarios from the benchmark using the following procedure:

- The only information that is modified from scenario to scenario is the ownership relation between edges and haulers, with pipes being randomly assigned to haulers.

- In order to get reasonably connected networks, the random assignment is not performed on individual pipes, but on some predetermined groups of pipes. More precisely, the pipes are divided in 16 groups, corresponding to the 16 Spanish autonomous communities (setting aside Canary Islands, which contain no pipes of the high-presure network).

- Then, each of the 16 groups is randomly assigned to one of the 7 available haulers. We keep the same number of haulers of the Spanish network which should provide enough richness to the random generating process (note that a hauler might end up with no assigned pipes in some realizations).

- This random process is repeated 10000 times, with the goal of obtaining very diverse realizations: homogeneous haulers, a single dominant hauler, split between medium haulers and small ones,...

- For each realization we obtain the resulting loss allocation for the five rules discussed in this paper. Finally, we compute the matrix of correlations between the allocations proposed by these five rules and also with the vector of the length of pipes owned by each hauler.[6]

In Tables 17.5, 17.6 and 17.7 we summarize the information contained in those correlation matrices. Table 17.5 contains the average of the 10000 correlation matrices obtained with the above procedure. As one might expect, all correlations are relatively high, with average numbers over 0.8 between all pairs of rules. The lowest number is found between the edge's rule and Shapley's rule but, more importantly, the highest average correlation is between Shapley's rule and the proportional tracing rule, reinforcing the observation in the analysis for the benchmark scenario. Indeed, this average correlation is almost perfect, being as high as 0.9933. The next highest correlations are found when comparing the flow rule with either the proportional tracing rule or the Shapley's rule, with values round 0.97. Although these correlations are also very high, they are significantly smaller than the previous one (0.9933 is just 0.007% away from perfect correlation, whereas 0.97 is more than 4 times further away).

Correlations	Flow	Aedge	Edge	Prop. Tracing	Shapley	Pipe Length
Flow	1.0000	0.9166	0.8717	0.9700	0.9707	0.6910
Aedge	0.9166	1.0000	0.8972	0.8988	0.8751	0.8154
Edge	0.8717	0.8972	1.0000	0.8776	0.8336	0.6200
Prop. Tracing	0.9700	0.8988	0.8776	1.0000	0.9933	0.6526
Shapley	0.9707	0.8751	0.8336	0.9933	1.0000	0.6446
Pipe Length	0.6910	0.8154	0.6200	0.6526	0.6446	1.0000

TABLE 17.5: Average of correlation matrices.

If we look now at Table 17.6, which contains, for each pair of rules, the minimum correlation between them across the 10000 realizations, we again see the strong connection between Shapley's rule and the proportional tracing rule. In the scenario where the correlation between them was smaller, it was

[6]We have also used other approaches to compare the different rules, all of them leading to the same qualitative results.

Correlations	Flow	Aedge	Edge	Prop. Tracing	Shapley	Pipe Length
Flow	1.0000	−0.1037	−0.2191	0.6020	0.5756	−0.8766
Aedge	−0.1037	1.0000	−0.3963	0.0743	−0.1729	−0.4711
Edge	−0.2191	−0.3963	1.0000	0.2929	0.0777	−0.6746
Prop. Tracing	0.6020	0.0743	0.2929	1.0000	0.9181	−0.8856
Shapley	0.5756	−0.1729	0.0777	0.9181	1.0000	−0.9824
Pipe Length	−0.8766	−0.4711	−0.6746	−0.8856	−0.9824	1.0000

TABLE 17.6: Minimum across correlation matrices.

Correlations	Flow	Aedge	Edge	Prop. Tracing	Shapley	Pipe Length
Flow	1.0000	1.0000	0.9998	0.9999	0.9999	0.9987
Aedge	1.0000	1.0000	0.9999	0.9998	0.9994	0.9997
Edge	0.9998	0.9999	1.0000	0.9996	0.9992	0.9995
Prop. Tracing	0.9999	0.9998	0.9996	1.0000	1.0000	0.9994
Shapley	0.9999	0.9994	0.9992	1.0000	1.0000	0.9998
Pipe Length	0.9987	0.9997	0.9995	0.9994	0.9998	1.0000

TABLE 17.7: Maximum across correlation matrices.

still over 0.9. For any other pair of rules, this number is at most 0.6 and in many cases it can even be negative. Finally, Table 17.7 contains the information about the maximum correlation between any pair of rules. Not surprisingly, this number is very close to one for every pair of rules.

Given the poor behavior observed by the Shapley value in the axiomatic analysis developed in Section 17.4, it is interesting to see that it exhibits such a high correlation with the proportional tracing rule which, arguably, may be considered the one performing better from the axiomatic point of view. We do not claim that the analysis we have just presented, based on numeric simulations, represents any kind of proof, but it suggests that there must be some mathematical connection between these two rules which might be the subject of future research.

17.7 Conclusions

In this paper we have studied the Shapley value in the context of loss allocation in energy networks and developed an axiomatic analysis to study its behavior with respect to different axioms. The main result in the paper, Proposition 1 shows that the behavior of the Shapley rule is far from being as good as that of other rules studied in the literature, such as the edge's rule and the proportional tracing rule. This leaves as an open problem the issue of finding new desirable properties that the Shapley rule might satisfy and which might ultimately lead to an axiomatic characterization.

Interestingly, we then develop a comparative analysis of the different allocation rules on a set of problems originated from real data and observe that the Shapley rule has a very high correlation (over 0.99) with the proportional tracing rule. This may seem a bit contradictory with the fact that these two rules exhibit a very different behavior with respect to the set of axioms discussed in Section 17.4. Then, an open question for future research would be to understand the mechanism driving this unusually high correlation.

Acknowledgements

The authors acknowledge support from Ministerio de Economía y Competitividad and FEDER through projects MTM2014-60191-JIN, ECO2014-52616-R, and ECO2015-70119-REDT and from Xunta de Galicia through projects INCITE09-207-064-PR, GRC 2015/014 and ED431C 2017/38. Gustavo Bergantiños acknowledges support from Fundación Séneca de la Región de Murcia through project 19320/PI/14. Ángel M. González-Rueda acknowledges support from Ministerio de Educación through Grant FPU13/01130.

Bibliography

[1] G. Bergantiños, J. González Díaz, A. M. González-Rueda, and M. P. Fernández de Córdoba. Loss allocation in energy transmission networks. *Games and Economic Behavior*, 102:69–97, 2017.

[2] G. Bergantiños and J. Vidal-Puga. A fair rule in minimum cost spanning tree problems. *Journal of Economic Theory*, 137:326–352, 2007.

[3] Boletín Oficial del Estado. Incentivo a la reducción de mermas en la red de transporte. In *Orden ITC/3128/2011*, volume 278. Spanish Government, 2011.

[4] Boletín Oficial del Estado. Coeficientes de mermas en las instalaciones gasistas. In *Orden IET/2446/2013*, volume 312. Spanish Government, December 2013.

[5] Boletín Oficial del Estado. Incentivo a la reducción de mermas en la red de transporte (amendment). In *Orden IET/2446/2013*, volume 312. Spanish Government, December 2013.

[6] British Petroleum. Bp statistical review of world energy. Annual report, 2013.

[7] Y. Chun. A pessimistic approach to the queueing problem. *Mathematical Social Sciences*, 51:171–181, 2006.

[8] Enerdata. Global energy statistical yearkbook. Annual report, 2017.

[9] E. Kalai and E. Zemel. Totally balanced games and games of flow. *Mathematics of Operations Research*, 7:476–478, 1982.

[10] A. Kar. Axiomatization of the Shapley value on minimum cost spanning tree games. *Games and Economic Behavior*, 38:265–277, 2002.

[11] S. C. Littlechild and G. Owen. A simple expression for the Shapley value in a special case. *Management Science*, 20:370–372, 1973.

[12] F. Maniquet. A characterization of the Shapley value in queueing problems. *Journal of Economic Theory*, 109:90–103, 2003.

[13] Regulation (EC). Common rules for the internal market in natural gas and repealing directive 98/30/ec. *Official Journal of the European Union (Legislation Series)*, 176:57–77, no. 55/2003.

[14] L. S. Shapley. A value for n-person games. In H. Kuhn and A. Tucker, editors, *Contributions to the theory of games II*, volume 28 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, 1953.