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for simple games with a priori unions

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Abstract: In this paper we define and axiomatically characterize an extension of the Deegan-Packel index for simple games with a priori unions. A real–world example illustrates this extension.

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1 Introduction

Power is an important concept in the study of simple games. Power indices are quantitative measures and indicators that characterize the game (see Aumann, 1977). In the literature we find several power indices: the Shapley–Shubik power index (Shapley and Shubik, 1954), the Banzhaf–Coleman power index (Banzhaf, 1965; Coleman, 1971), the Deegan–Packel power index (Deegan and Packel, 1978), the Johnston power index (Johnston, 1978), and the Public Good Index (Holler, 1982). These traditional power indices depend on the characteristic function of the game only. Their strength is to provide a measure of the probability that an actor in a simple game might be pivotal in a vote, in the sense that he might be able to transform a losing coalition into a winning one (or a special type of losing coalition into a special type of winning one) and viceversa. Following Aumann (1977, p. 464), we could say that "none of them is the *right* solution concept; they are all indicators, not predictors. Different solution concepts are like different indicators of an economy; different methods for calculating a price index; different maps (road, political, geologic, etc., not to speak of scale, projection, etc.); different stock indices (Dow Jones, etc.) They depict or illuminate the situation from different angles; each one stresses certain aspects at the expense of others".

For representing social decision situations adequately, sophisticated models have been developed. One of them is the TU game endowed with a priori unions, that is, a partition of the player set which describes a pre-defined (exogenously given) coalition structure. The traditional power indices are not suitable for measuring the distribution of power in these situations because adequate measures of power should take the coalition structure into account.

The so–called Owen value (Owen, 1977), is an extension of the Shapley value to social decisions with a priori unions. As a first step this measure splits the total amount among the unions, according to the Shapley value, in the induced game played by the unions (quotient game). Then once again using the Shapley value within each union, total reward are allocated among its members (quotient game property), taking into account the potential of their joining other unions. The Owen value uses the Shapley value in both processes. The Owen value is a coalitional value of Shapley, in the sense that it coincides with the Shapley value when the system of unions is such that each union is a singleton. One of the most compelling properties of the Owen value is the property of symmetry in the quotient game: given two unions which play symmetric roles[5] in the quotient game, they are awarded with the same apportionment of the total payoff.

Using similar reasoning, Owen (1982) proposed an extension of the Banzhaf value to the framework of TU games with a priori unions: the Banzhaf–Owen value. Here the assessments in each step are given by the Banzhaf value. This value or its restriction to simple games have been axiomatically characterized, by Albizuri (2001), by Amer *et al.* (2002) and by Alonso–Meijide *et al.* (2007). It is interesting to note that neither the property of symmetry in the quotient game nor the quotient game property are satisfied by this measure.

Alonso–Meijide and Fiestras–Janeiro (2002) proposed a new value for TU games with a priori unions which is an extension of the Banzhaf value and satisfies the already mentioned properties of quotient game and symmetry in the quotient game: the symmetric coalitional Banzhaf value. It reflects the result of a bargaining procedure by which the a priori unions receive in the quotient game the payoff given by the Banzhaf value and within each union the original players share the value of the union among themselves by using the Shapley value.

In this paper we will extend the Deegan–Packel index to simple games with a priori unions. We define and characterize an extension such that the corresponding coalitional power index coincides with the Deegan–Packel index when each union is formed by only one player or there is only one union. This extension satisfies two properties of symmetry, one among players of the same union, and the second one among unions in the game played among unions. Besides, this extension satisfies the compelling quotient game property.

The paper is organized as follows. In Section 2, we recall some basic definitions. In Section 3, we introduce the modification of the Deegan–Packel power index. In Section 4, we characterize this modification. In Section 5, we illustrate this extension with a real–world example. We conclude with some remarks.

2 Preliminaries

2.1 Simple Games

Simple games are commonly used to represent decision-making processes, that is, the voting body and the decision-making rules. A **simple game** is

[5]Later on, we will detail the exact meaning of this symmetry.

a pair (N, W) with N a finite set and W a family of subsets of N satisfying:

- $N \in W$ and
- the monotonicity property, *i.e.*,

$$S \subseteq T \subseteq N \text{ and } S \in W \text{ implies } T \in W.$$

This representation of simple games follows Peleg and Sudhölter (2003). We denote by $SI(N)$ the set of simple games with player set N. In a simple game (N, W), a coalition $S \subseteq N$ is **winning** if $S \in W$ and is **losing** if $S \notin W$, W being the set of winning coalitions. Intuitively, N is the set of members of a committee and W is the set of coalitions that fully control the involved decision problem. Parliaments, town councils, and the UN Security Council are examples for such committees.

A coalition $S \in W$ is a **minimal winning** coalition if every proper subset of S is a losing coalition, that is, S is a minimal winning coalition in (N, W) if $S \in W$ and $T \notin W$ for any $T \subset S$. We denote by M^W the set of minimal winning coalitions[6] of the simple game (N, W).

A **null player** in a simple game (N, W) is a player i such that $i \notin S$ for all $S \in M^W$. Two players $i, j \in N$ are **symmetric** in a simple game (N, W) when $S \cup i \in W$ if and only if $S \cup j \in W$[7]. Given a player $i \in N$ we denote by M_i^W the set of minimal winning coalitions such that i belongs to, that is, $M_i^W = \left\{S \in M^W / i \in S\right\}$. Given a coalition $T \subseteq N$, the unanimity game of T, (N, W_T), is the simple game with $M^{W_T} = \{T\}$.

A **power index** is a function f which assigns to a simple game (N, W) an n-dimensional real vector $f(N, W)$, where the i-th component of this vector $f_i(N, W)$ is the power of player i in the game (N, W) according to f.

2.2 The Deegan-Packel index

The Deegan–Packel power index (Deegan and Packel, 1978) gives a measure of power that satisfies certain conditions: Minimality (only minimal winning coalitions will emerge victorious), equiprobability (each minimal winning coalition has an equal probability of forming), and solidarity (players in a minimal winning coalition divide the power equally). These conditions seem

[6]The traditional von Neumann–Morgenstern notation uses W^m for the set of minimal winning coalitions of the simple game. Here M^W is more convenient.

[7]For convenience we write $S \cup i$ and $S \backslash i$ instead of $S \cup \{i\}$ and $S \backslash \{i\}$, respectively. Moreover, the number of elements of a set A will be $\mid A \mid$.

reasonable in many cases. All rational players want to maximize power and then, only minimal winning coalitions are formed (minimality). All minimal winning coalitions (equiprobability) and all players inside each minimal winning coalition (solidarity) play the same role.

The Deegan–Packel power index of a player i in the simple game (N, W) is given by:

$$\rho_i (N, W) = \frac{1}{|M^W|} \sum_{S \in M_i^W} \frac{1}{|S|}. \tag{1}$$

With the Deegan-Packel index, players should look for minimal winning coalitions that are minimal in the cardinality of the number of players. A different index based on minimal winning coalitions is the Public Good Index (Holler, 1982). For this index, the size of the minimal winning coalition does not matter to measure power.

It is obvious that for unanimity games this index coincides with the Shapley–Shubik power index. In Deegan and Packel (1978) a probabilistic interpretation of this index and a characterization are given. In this characterization the properties of symmetry, efficiency, null player and a new property called mergeability are used.

Mergeable games. Two simple games (N, W) and (N, V) are mergeable if for any $S \in M^W$ and for any $T \in M^V$, $S \not\subseteq T$ and $T \not\subseteq S$.

Given two simple games $(N, W), (N, V)$, the simple game $(N, W \vee V)$ is defined in such a way that a coalition $S \in W \vee V$ if $S \in W$ or $S \in V$.

Mergeability condition. The mergeability condition guarantees that the set of minimal winning coalitions of the game $(N, W \vee V)$, where (N, W) and (N, V) are two mergeable games, is the union of the minimal winning coalition sets of the games (N, W) and (N, V) *i.e.*, $\left|M^{W \vee V}\right| = \left|M^W\right| + \left|M^V\right|$ if (N, W) and (N, V) are mergeable games.

Mergeability. A power index f satisfies mergeability if for any pair of mergeable games $(N, W), (N, V)$, it holds that

$$f (N, W \vee V) = \frac{\left|M^W\right| f (N, W) + \left|M^V\right| f (N, V)}{\left|M^{W \vee V}\right|}. \tag{2}$$

Deegan and Packel (1978) characterized ρ as follows.

- The unique power index f on $SI(N)$ satisfying mergeability, null player, symmetry, and efficiency is the Deegan-Packel power index.

Alternatively, Lorenzo-Freire *et al.* (2007) characterized the Deegan-Packel index replacing the property of mergeability with the property of minimal monotonicity.

Minimal monotonicity. A power index f satisfies minimal monotonicity if for any pair of simple games (N,W), (N,V), it holds that for all player $i \in N$ such that $M_i^W \subseteq M_i^V$,

$$f_i(N,V)\left|M^V\right| \geq f_i(N,W)\left|M^W\right|,$$

i.e., if the set of minimal winning coalitions containing a player $i \in N$ in game (N,W) is a subset of minimal winning coalitions containing this player in game (N,V), then the power of player i in game (N,V) is not less than power of player i in game (N,W) (first, this power must be normalized by the number of minimal winning coalitions in games (N,W) and (N,V), respectively).

- The unique power index f on $SI(N)$ satisfying minimal monotonicity, null player, symmetry, and efficiency is the Deegan-Packel power index.

2.3 Games with a priori unions

Given N, we will denote by $P(N)$ the set of all partitions of N. An element $P \in P(N)$ is called a coalition structure (formed by a priori unions on N). A simple game with a coalition structure is a triple (N,W,P), where (N,W) is a simple game and $P \in P(N)$. We denote by SIU the set of simple games with a priori unions and by $SIU\,(N)$ the set of simple games with a priori unions and player set N.

Given $(N,W,P) \in SIU\,(N)$, with $P = \{P_1, P_2, \ldots, P_u\}$, the quotient game is the simple game $\left(U, \overline{W}\right)$, where the set of players is specified by the set of index numbers $U = \{1, 2, \ldots, u\}$ that represent the a priori unions contained in P. A coalition $R \subseteq U$ is a winning coalition in $\left(U, \overline{W}\right)$ if $\bigcup\limits_{k\in R} P_k$ is a winning coalition in (N,W). Two unions $P_k, P_s \in P$ are symmetric if k and s are symmetric players in $\left(U, \overline{W}\right)$.

We must introduce some further notation. Given a game with a priori unions (N,W,P), with $M^W = \{S_1, S_2, \ldots, S_l\}$, $P = \{P_1, P_2, \ldots, P_u\}$, and the set $U = \{1, 2, \ldots, u\}$:

- Two trivial systems of unions are given by $P^n = \{\{1\}, \{2\}, \ldots, \{n\}\}$ and $P^N = \{N\}$.
- The representatives of a coalition $S \subseteq N$ in the quotient game $\left(U, \overline{W}\right)$ form a coalition $u\,(S) \subseteq U$, where $j \in u\,(S)$ if and only if there exists a player i of P_j such that $i \in S$. In this way, $u(S)$ is defined as a set of

index numbers that characterize the a priori unions that are involved in the forming of S, that is,

$$u(S) = \{j \in U / P_j \cap S \neq \emptyset\}.$$

- $M^{\overline{W}}$ denotes the set of minimal winning coalitions in the quotient game, that is,

$$M^{\overline{W}} = \left\{ R \subseteq U / R \in \overline{W} \text{ and } R' \notin \overline{W} \text{ for all } R' \subset R \right\}.$$

- Given an a priori union $k \in U$, $M_k^{\overline{W}}$ denotes the set of minimal winning coalitions in the quotient game such that k belongs to them, that is,

$$M_k^{\overline{W}} = \left\{ L \in M^{\overline{W}} / k \in L \right\}.$$

Given a simple game with a priori unions (N, W, P), taking into account that a negotiation among unions will happen, we will say that a coalition $S \in M^W$ is **irrelevant** if $u(S) \notin M^{\overline{W}}$, that is, a irrelevant coalition is a minimal winning coalition in the game (N, W) such that its representatives in the quotient game do not constitute a minimal winning coalition in $(U, \overline{W})$.

With respect to the negotiation among players belonging to the same union, one of the main assumptions in Owen (1977) is that every coalition $S \subseteq P_k$ has the possibility of forming a winning coalition joining with one or more of the remaining unions different from P_k. But the Owen value does not consider the possibility of a coalition among a subset S and "proper" subsets of other unions. Taking into account this limitation with respect to the formation of winning coalitions, we introduce the concept of essential coalitions.

Given a simple game with a priori system of unions (N, W, P), we will say that a coalition $\emptyset \neq S \subseteq P_k$ is **essential** with respect to $R \in M^{\overline{W}}$ if $k \in R$, $S \cup \left(\cup_{l \in R \setminus k} P_l\right) \in W$, and $T \cup \left(\cup_{l \in R \setminus k} P_l\right) \notin W$, for every $\emptyset \neq T \subset S$. Given an union P_k, $E^{R,k}(N, W, P)$ denotes the set of essential coalitions of the game (N, W, P) with respect to R and $E_i^{R,k}(N, W, P)$ denotes the subset of $E^{R,k}(N, W, P)$ formed by coalitions S, such that $i \in S$. $E(N, W, P)$ denotes the set of essential coalitions of the game (N, W, P).

Example 1 *Take a simple game with a priori unions (N, W, P) with $N = \{a, b, c, d, e, f\}$, $P = \{P_1, P_2, P_3\}$ where $P_1 = \{a\}, P_2 = \{b, c, d\}$ and $P_3 = \{e, f\}$ and $M^W = \{S_1, S_2, S_3, S_4, S_5\}$ where $S_1 = \{a, b\}, S_2 = \{a, c, d\}, S_3 =$*

$\{a,e,f\}$, $S_4=\{a,c,e\}$, and $S_5=\{b,c,d,e,f\}$. The minimal winning coalitions in the quotient game $\left(U,\overline{W}\right)$ are:

$$M^{\overline{W}}=\{\{1,2\},\{1,3\},\{2,3\}\},$$

where $U=\{1,2,3\}$.

In this game, $S_4=\{a,c,e\}$ is a minimal winning coaliton. However, its coalition of representatives $u(S_4)=\{1,2,3\}$ is not minimal in the quotient game. Hence, S_4 is irrelevant.

An essential coalition with respect to $R=\{1,2\}$ is given by $\{a\}$.

3 The coalitional Deegan–Packel index

In the context of simple games with a priori unions, **a coalitional power index** is a function f which assigns an n-dimensional real vector $f\left(N,W,P\right)$ to a simple game with a priori unions (N,W,P), where the i-th component of this vector $f_i\left(N,W,P\right)$ is the power of player i in the game $\left(N,W,P\right)$ according to f.

To discuss the coalitional Deegan-Packel index, we introduce two modifications of the minimality condition, of the equiprobability condition and of the solidarity condition. In the definitions of the modified conditions, we consider the two levels of negotiation, (a) among unions, and (b) among players belonging to the same a priori union. These conditions are:

- **Minimality in the quotient game:** Only minimal winning coalitions that imply a minimal winning coalition in the quotient game are taken into account.

- **Equiprobability in the quotient game**: Each minimal winning coalition in the quotient game has an equal probability of forming.

- **Solidarity in the quotient game:** Unions in a winning coalition of the quotient game divide the spoils equally.

- **Minimality inside unions:** Only essential coalitions will emerge victorious.

- **Equiprobability inside unions:** Essential coalitions with respect to the same minimal winning coalition in the quotient game, have an equal probability of forming.

- **Solidarity inside unions:** Players of the same a priori union in an essential coalition divide the spoils equally.

The meaning of these conditions is illustrated in the following example.

Example 2 *Take the simple game with a priori unions of Example 1. By the condition of minimality in the quotient game, the minimal winning coalition S_4 will not emerge victorious because its representative in the quotient game, $\{1,2,3\}$, is not a minimal winning coalition in this game.*

By the condition of equiprobability in the quotient game, $\{1,2\}, \{1,3\}$, and $\{2,3\}$ have an equal probability of forming.

Finally, by the condition of equiprobability inside unions, S_1 and S_2 are equiprobable, because its set of representatives in the quotient game is $\{1,2\}$.

We define a new power index, that we will name the coalitional Deegan–Packel index. This index is consistent with the previous conditions.

Definition 1 *Given $(N,W,P) \in SIU(N)$, the coalitional Deegan–Packel index of a player $i \in P_k$ is:*

$$\Lambda_i(N,W,P) = \frac{1}{\left|M^{\overline{W}}\right|} \sum_{R \in M_k^{\overline{W}}} \frac{1}{|R|} \frac{1}{\left|E^{R,k}(N,W,P)\right|} \sum_{L \in E_i^{R,k}(N,W,P)} \frac{1}{|L|}. \tag{3}$$

It is easy to prove that this index is an extension of the Deegan-Packel index. The coalitional Deegan–Packel index coincides with the Deegan-Packel index when the system of unions is either of the trivial ones, P^n or P^N.

The index Λ is consistent with the previous conditions. Only minimal winning coalitions that give rise to a minimal winning coalition in the quotient game have influence (minimality in the quotient game). Taking into account the amount $\left|M^{\overline{W}}\right|^{-1}$, all of them are treated in an equiprobable way (equiprobability in the quotient game). For each coalition $R \in M^{\overline{W}}$, the term $|R|^{-1}$ indicates that a priori unions belonging to a winning coalition in the quotient game play identical roles (solidarity in the quotient game). Only essential coalitions have influence in the division of power (minimality inside unions). We consider essential coalitions only because the " surplus players" do not get a share. Given a coalition $R \in M^{\overline{W}}$ and an union P_k, all essential coalitions with respect to R are dealt with in the same way, $\left|E^{R,k}(N,W,P)\right|^{-1}$ (equiprobability inside unions). Finally, the term $|L|^{-1}$

assures that the payoff for a player i is the same as for the others $|L|-1$ players of an essential coalition L (solidarity inside unions).

The index Λ has a probabilistic interpretation. Take a player $i \in P_k$, such that $i \in L$, where L is an essential coalition with respect to the a priori union P_k and the minimal winning coalition in the quotient game $R \in M^{\overline{W}}$. Assume that the power assigned to player i according to L is $|R|^{-1}|L|^{-1}$ when coalition L forms. If an essential coalition L with respect to P_k and R has a probability $\left|M^{\overline{W}}\right|^{-1}\left|E^{R,k}(N,W,P)\right|^{-1}$ of forming, then the mean of the power assigned to player i is precisely Λ_i.

4 Axiomatic characterization of the coalitional Deegan–Packel index

For any power index, understood as a solution concept for simple games, it is always interesting, in both theory and practice, to have an explicit formula and a list of properties of the index, "as long as possible".

Besides, a set of basic (and assumed independent and hence minimal) properties is a most convenient and economic tool to decide on the use of the index. Finally, such a set allows a researcher to compare a given value with others and select the most suitable one for the problem he or she is facing each time. Aumann (1977, p. 471) wrote "axiomatizations serve a number of useful purposes. First, like any other alternative characterization, they shed additional light on a concept, enable us to understand it better. Second, they underscore and clarify important similarities between concepts, as well as differences between them". Obviously, there is no power index able to cover all situations. Perhaps only a few properties, found in the literature, can really be considered absolutely compelling (for instance symmetry properties), but even those that appear as most promising in this sense might well be conditioned by the characteristics of the problem to which we apply the index they define. The conclusion is to look at axioms with an open mind and without a priori value judgements.

First of all, we recall the well-known properties of efficiency, null player, symmetry inside unions, symmetry among unions, and quotient game property of a coalitional power index.

Efficiency. A power index f satisfies efficiency if and only if for every $(N,W,P) \in SIU(N)$, $\sum_{i\in N} f_i(N,W,P) = 1$.

Null Player. A power index f satisfies null player if and only if for every $(N,W,P) \in SIU(N)$ and $i \in N$ a null player in the game (N,W), then $f_i(N, W, P) = 0$.

Symmetry inside unions. A power index f satisfies symmetry inside unions if and only if for every $(N, W, P) \in SIU(N)$, and $i, j \in P_k$ symmetric players in the game (N, W), $f_i(N, W, P) = f_j(N, W, P)$.

Symmetry among unions. A power index f satisfies symmetry among unions if and only if for every $(N, W, P) \in SIU(N)$, and $l, k \in U$ symmetric players in the quotient game, then

$$\sum_{i \in P_k} f_i(N, W, P) = \sum_{i \in P_l} f_i(N, W, P).$$

Quotient game property. A power index f satisfies the quotient game property if and only if for every $(N, W, P) \in SIU(N)$, and $k \in U$, then

$$f_k(U, \overline{W}, P^u) = \sum_{i \in P_k} f_i(N, W, P).$$

If two games (N, W, P) and (N, V, P) are **mergeable** in the quotient game, the mergeability condition guarantees that $\left|M^{\overline{W} \vee \overline{V}}\right| = \left|M^{\overline{W}}\right| + \left|M^{\overline{V}}\right|$. The property of mergeability in the quotient game states that power in a merged game is a weighted mean of power of the two component games, with the number of minimal winning coalitions in the quotient of each component game providing the weights.

Mergeability in the quotient game. A power index f satisfies mergeability in the quotient game if for any pair $(N, W, P), (N, V, P)$, such that $(U, \overline{W})$ and $(U, \overline{V})$ are mergeable, it holds that

$$f\left(N, W \vee V, P\right) = \frac{\left|M^{\overline{W}}\right| f\left(N, W, P\right) + \left|M^{\overline{V}}\right| f\left(N, V, P\right)}{\left|M^{\overline{W} \vee \overline{V}}\right|}.$$

The property of mergeability inside unions states that power in a merged game is a weighted mean of power of the two component games, with the number of minimal winning coalitions of each component game providing the weights, when all the minimal winning coalitions of the merged games are included in the same a priori union. Notice that in such a case there is only one minimal winning coalition in the quotient game.

Mergeable games inside unions. Two simple games with a priori unions (N, V, P) and (N, W, P) are mergeable inside unions if:

- (N, W) and (N, V) are mergeable,

- there exists $k \in U$ such that for every $S \in M^W \cup M^V$ it holds $S \subset P_k$.

Mergeability inside unions. A power index f satisfies mergeability inside unions if for any pair $(N, W, P), (N, V, P)$ of mergeable inside unions simple games with a priori unions then,

$$f(N, W \vee V, P) = \frac{\left|M^W\right| f(N, W, P) + \left|M^V\right| f(N, V, P)}{\left|M^{W \vee V}\right|}.$$

Invariance with respect to essential coalitions inside unions says that in a simple game with a priori unions in which the same a priori unions are represented in all the minimal winning coalitions, the power of players does not change if the set of minimal winning coalitions changes but it coincides with the original set of essential coalitions.

Invariance with respect to essential coalitions inside unions. A power index f satisfies invariance with respect to essential coalitions inside unions if for any pair $(N, W, P), (N, V, P)$, such that $E(N, W, P) = E(N, V, P)$, $u(S) = R$, for every $S \in M^W \cup M^V$, then $f(N, W, P) = f(N, V, P)$.

Independence of irrelevant coalitions among unions says that elimination of irrelevant coalitions of the game will not change the power of the players.

Independence of irrelevant coalitions among unions. A power index f satisfies independence of irrelevant coalitions among unions if for any (N, W, P), given $S \in M^W$ such that $u(S) \notin M^{\overline{W}}$, then $f(N, W, P) = f(N, W', P)$ where $M^{W'} = M^W \setminus S$.

Theorem 1 *The coalitional Deegan-Packel index is the unique power index which satisfies the properties of efficiency, null player, symmetry inside unions, symmetry among unions, mergeability in the quotient game, mergeability inside unions, invariance with respect to essential coalitions inside unions, and independence of irrelevant coalitions among unions.*

Proof.

Existence. Let (N, W, P) be a simple game with a priori unions where $P = \{P_1, \cdots, P_u\}$ and $U = \{1, \cdots, u\}$. We prove that the coalitional Deegan–Packel power index Λ satisfies the above properties.

If $i \in N$ is a *null player*, $E_i^{R,k}(N, W, P) = \emptyset$ for all $R \in M_k^{\overline{W}}$ where $i \in P_k$, and then $\Lambda_i(N, W, P) = 0$.

If two players $i, j \in P_k$ are symmetric, then the sets $E_i^{R,k}(N, W, P)$ and $E_j^{R,k}(N, W, P)$ are symmetric in the same way. Therefore, the coalitional index Λ satisfies the property of *symmetry inside unions.*

Given a union $P_k \in P$, it holds that

$$\sum_{i \in P_k} \Lambda_i(N, W, P)$$

$$= \sum_{i \in P_k} \frac{1}{\left| M^{\overline{W}} \right|} \sum_{R \in M_k^{\overline{W}}} \frac{1}{|R|} \frac{1}{|E^{R,k}(N, W, P)|} \sum_{L \in E_i^{R,k}(N,W,P)} \frac{1}{|L|}$$

$$= \frac{1}{\left| M^{\overline{W}} \right|} \sum_{R \in M_k^{\overline{W}}} \frac{1}{|R|} \frac{1}{|E^{R,k}(N, W, P)|} \sum_{i \in P_k} \sum_{L \in E_i^{R,k}(N,W,P)} \frac{1}{|L|}$$

$$= \frac{1}{\left| M^{\overline{W}} \right|} \sum_{R \in M_k^{\overline{W}}} \frac{1}{|R|}.$$

That is, the value of the solution Λ assigned to an union $P_k \in P$ coincides with the Deegan–Packel power index of the player $k \in U$ in the quotient game. As the Deegan–Packel power index is symmetric, if two unions $P_k, P_j \in P$ are symmetric in the quotient game, it holds that

$$\sum_{i \in P_k} \Lambda_i(N, W, P) = \sum_{i \in P_j} \Lambda_i(N, W, P),$$

and thus, the index Λ satisfies the property of *symmetry among unions*.

From above, it follows that the coalitional Deegan–Packel index satisfies the quotient game property. Moreover, we know that the Deegan–Packel power index is efficient. Then, we get that the coalitional Deegan–Packel index is *efficient*.

The index Λ satisfies the property of *mergeability in the quotient game* because if two games $(N, W, P), (N, V, P) \in SIU(N)$ are mergeable in the quotient game, it holds that for any player $i \in P_k$,

$$\Lambda_i(N, W \vee V, P)$$

$$= \frac{1}{\left| M^{\overline{W \vee V}} \right|} \sum_{R \in M_k^{\overline{W \vee V}}} \frac{1}{|R|} \frac{1}{|E^{R,k}(N, W \vee V, P)|} \sum_{L \in E_i^{R,k}(N,W \vee V,P)} \frac{1}{|L|}$$

$$= \frac{1}{\left| M^{\overline{W \vee V}} \right|} \sum_{R \in M_k^{\overline{W}}} \frac{1}{|R|} \frac{1}{|E^{R,k}(N, W, P)|} \sum_{L \in E_i^{R,k}(N,W,P)} \frac{1}{|L|}$$

$$+ \frac{1}{\left| M^{\overline{W \vee V}} \right|} \sum_{R \in M_k^{\overline{V}}} \frac{1}{|R|} \frac{1}{|E^{R,k}(N, V, P)|} \sum_{L \in E_i^{R,k}(N,V,P)} \frac{1}{|L|}$$

$$= \frac{\left|M^{\overline{W}}\right| \Lambda_i(N,W,P) + \left|M^{\overline{V}}\right| \Lambda_i(N,V,P)}{\left|M^{\overline{W \vee V}}\right|}.$$

Given two games $(N,W,P),(N,V,P) \in SIU(N)$ are mergeable inside unions, it holds that $M^{\overline{W}} = M^{\overline{V}} = M^{\overline{W\vee V}} = \{R\}$ for some $R = \{k\}$ with $k \in U$.

Besides, it holds that $\mid E^{R,k}(N,W,P) \mid = \mid M^W \mid$, $\mid E^{R,k}(N,V,P) \mid = \mid M^V \mid$, and $\mid E^{R,k}(N,W \vee V,P) \mid = \mid M^{W\vee V} \mid$.

Then, for $i \in P_k$,

$$\Lambda_i\,(N,W \vee V,P)$$

$$= \frac{1}{\left|M^{\overline{W\vee V}}\right|}\frac{1}{|R|}\frac{1}{|M^{W\vee V}|}\sum_{L\in M_i^{W\vee V}}\frac{1}{|L|}$$

$$= \frac{\left|M^W\right|}{|M^{W\vee V}|}\left(\frac{1}{\left|M^{\overline{W}}\right|}\frac{1}{|R|}\frac{1}{|M^W|}\sum_{L\in M_i^W}\frac{1}{|L|}\right) + \frac{\left|M^V\right|}{|M^{W\vee V}|}\left(\frac{1}{\left|M^{\overline{V}}\right|}\frac{1}{|R|}\frac{1}{|M^V|}\sum_{L\in M_i^V}\frac{1}{|L|}\right)$$

$$= \frac{\left|M^W\right| \Lambda_i(N,W,P) + \left|M^V\right| \Lambda_i(N,V,P)}{|M^{W\vee V}|}.$$

For $i \notin P_k$, mergeability inside unions is given because of

$$\Lambda_i(N,W \vee V,P) = \Lambda_i(N,W,P) = \Lambda_i(N,V,P) = 0.$$

We conclude that the coalitional Deegan-Packel index satisfies the property of *mergeability inside unions*.

If two games (N,W,P) and (N,V,P) are in the conditions of the *invariance with respect to essential coalitions inside unions* it holds that

$$\mid M^{\overline{W}} \mid = \mid M^{\overline{V}} \mid \ \text{ and } E(N,W,P) = E(N,V,P).$$

Thus, $\Lambda(N,W,P) = \Lambda(N,V,P)$.

The index Λ satisfies the property of *independence of irrelevant coalitions among unions*. Take $(N,W,P) \in SIU(N)$ such that there exists $S \in M^W$ with $u(S) \notin M^{\overline{W}}$. We consider the game $(N,V,P) \in SIU(N)$ where $M^V = M^W \backslash S$ it holds that $M^{\overline{W}} = M^{\overline{V}}$ and $E(N,W,P) = E(N,V,P)$. Then, by Eq. 3, $\Lambda_i(N,W,P) = \Lambda_i(N,V,P)$.

Uniqueness. Let us take a power index f which satisfies all the above properties. Let us take $(N, W, P) \in SIU(N)$ with $M^W = \{S_1, \ldots, S_l\}$. Since the power index f satisfies *independence of irrelevant coalitions among unions* we can assume that

$$u(S) \in M^{\overline{W}}, \text{ for every } S \in M^W. \tag{4}$$

First, we assume that $l = 1$. In that case (N, W) is a unanimity game, for instance, (N, W_S) with $S \subset N$. Since f satisfies *efficiency, null player, symmetry inside unions, and symmetry among unions*

$$f_i(N, W_S, P) = \begin{cases} \frac{1}{|u(S)|}\frac{1}{|S \cap P_k|} & \text{if } i \in S \cap P_k \\ 0 & \text{if } i \notin S. \end{cases},$$

In this case, f_i coincides with Λ_i for every $i \in N$, in all unanimity games.

Let us assume that $l > 1$. We consider the partition of M^W, $\{T_1, \ldots, T_r\}$ which is defined in such a way that two coalitions $S_j, S_p \in T_h$ if and only if $u(S_j) = u(S_p)$ with $h = 1, 2, \ldots, r$. It holds that $W = W^{T_1} \vee W^{T_2} \vee \ldots \vee W^{T_r}$ where $W^{T_h} = W_{S_{h_1}} \vee W_{S_{h_2}} \vee \ldots \vee W_{S_{h_{t_h}}}$ with $T_h = \left\{S_{h_1}, S_{h_2}, \ldots, S_{h_{t_h}}\right\} \subseteq M^W$ for every $h \in \{1, 2, \ldots, r\}$. Notice that the games $\left(N, W^{T_j}, P\right)$ and $\left(N, W^{T_p}, P\right)$ for $j, p = 1, 2, \ldots, r$, $(j \neq p)$ are mergeable in the quotient game because $M^{\overline{W^{T_j}}} \neq M^{\overline{W^{T_p}}}$ and $\left|M^{\overline{W^{T_j}}}\right| = \left|M^{\overline{W^{T_p}}}\right| = 1$. Then, by the property of *mergeability in the quotient game*, it holds that:

$$\begin{aligned} f_i(N, W, P) &= \frac{1}{\left|M^{\overline{W}}\right|} \sum_{h=1}^{r} \left|M^{\overline{W^{T_h}}}\right| f_i\left(N, W^{T_h}, P\right) \\ &= \frac{1}{\left|M^{\overline{W}}\right|} \sum_{h=1}^{r} f_i\left(N, W^{T_h}, P\right). \end{aligned} \tag{5}$$

Take $h \in \{1, \ldots, r\}$, $R = u(S_j)$, for every $S_j \in T_h$, and the game (N, W_h, P) with $W_h = \vee_{k \in R} W_{T_h}^{R,k}$ where

$$M^{W_{T_h}^{R,k}} = E^{R,k}(N, W_{T_h}, P). \tag{6}$$

Then, $M^{W_h} = E(N, W_h, P)$, $E\left(N, W^{T_h}, P\right) = E(N, W_h, P)$, and $u(S) = R$ for every $S \in M^{W^{T_h}}$. Since f satisfies *invariance with respect to essential coalitions inside unions*, $f_i(N, W^{T_h}, P) = f_i(N, W_h, P)$, for every $i \in N$. Furthermore, any pair of simple games with a priori unions in the collection

$\Big\{(N, W_{T_h}^{R,k}, P) \,/\, k \in R\Big\}$ are mergeable in the quotient game. Then, for each $i \in P_k$,

$$|R| f_i(N, W^{T_h}, P) = |R| f_i(N, W_h, P) = f_i(N, W_{T_h}^{R,k}, P), \tag{7}$$

because f satisfies *null player* property and $i \in P_k$ is a null player in every simple game $(N, W_{T_h}^{R,j}, P)$ with $j \neq k$. Notice that $W_{T_h}^{R,k} = W_{L_1} \vee \ldots \vee W_{L_s}$ with $L_j \subset P_k$ and (N, W_{L_1}), …,(N, W_{L_s}), being mergeable games. Since f satisfies *mergeability inside unions*, then

$$|M^{W_{T_h}^{R,k}}| f_i(N, W_{T_h}^{R,k}, P) = \sum_{L_j \in M_i^{W_{T_h}^{R,k}}} f_i(N, W_{L_j}, P). \tag{8}$$

Using the definition of the power index for unanimity games, Equations (6)-(8) imply

$$f_i(N, W^{T_h}, P) = \frac{1}{|R|} \frac{1}{|E^{R,k}(N, W, P)|} \sum_{L_j \in E^{R,k}(N,W,P)_i} \frac{1}{|L_j|}. \tag{9}$$

Replacing Eq. 9 in Eq. 5, finishes the proof. □

5 An Example

We compute the coalitional Deegan– Packel index to analyze the Parliament of Catalonia which has been arisen from the election held on November 1^{st}, 2006. This Parliament has also been studied in Carreras *et al.* (2006)[8]. The Parliament of Catalonia consists of 135 members. Following these elections, the Parliament was composed of:

1. 48 members of *CIU*, *Convergéncia i Unió*, a Catalan nationalist middle-of-the-road party,

2. 37 members of *PSC*, *Partido de los Socialistas de Cataluña*, a moderate left-wing socialist party federated to the *Partido Socialista Obrero Español*,

3. 21 members of *ERC*, *Esquerra Republicana de Cataluña*, a radical Catalan nationalist left-wing party,

[8] Carreras *et al.* used binomial semivalues to explain the behavior of one of the parties (ERC).

4. 14 members of PPC, *Partido Popular de Cataluña*, a conservative party which is a Catalan delegation of the *Partido Popular*,

5. 12 members of ICV, *Iniciativa por Cataluña-Los Verdes-Izquierda Alternativa*, a coalition of ecologist groups and Catalan eurocommunist parties federated to *Izquierda Unida*, and

6. 3 members of $C's$, *Ciudadanos-Partidos de la Ciudadanía*, a non-Catalanist party.

This Parliament can be represented as the following weighted majority game

$$v = [68 : 48, 37, 21, 14, 12, 3].$$

For the sake of clarity we identify CIU as player 1, PSC as player 2, ERC as player 3, PPC as player 4, ICV as player 5 and $C's$ as player 6. Then, taking $N = \{1,2,3,4,5,6\}$, the corresponding minimal winning coalitions are:

$$W = \{\{1,2\}, \{1,3\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\}\}.$$

We see that members of the $C's$ party are null players. Two main aspects characterized politics in Catalonia: the left to right axis and the Spanish centralism to Catalanism axis. Taking into account this fact we consider two possible systems of a priori unions:

$$P^1 = \{\{1\}, \{2\}, \{3,5\}, \{4\}, \{6\}\},$$

$$P^2 = \{\{1\}, \{2,3,5\}, \{4\}, \{6\}\}.$$

In Table 1, we present the Deegan–Packel power index and the coalitional Deegan–Packel index for the two systems of unions.

The system of unions P^1 takes into account the left-wing and the Catalanism aspects of ERC and ICV. For P^1, the coalitional Deegan–Packel index assigns a non-zero value to CIU because in the game played by unions there are some minimal winning coalitions which involve union $\{1\}$. These coalitions are $\{\{1\}, \{2\}\}$ and $\{\{1\}, \{3,5\}\}$. If we consider the system of unions P^2, then the union $\{2,3,5\}$ has veto power in the game played by unions and players outside $\{2,3,5\}$ get value zero.

Party	Shares of seats	ρ	$\Lambda\left(P^1\right)$	$\Lambda\left(P^2\right)$
CIU	0.3556	0,2667	0.3333	0
PSC	0.2741	0.2333	0.3333	0.333
ERC	0.1037	0.2333	0.25	0.333
PPC	0.1155	0.1333	0	0
IC	0.0889	0.1333	0.0834	0.333
C's	0.0222	0	0	0

Table 1: Some power indices in the Catalonian Parliament November 2006.

6 Final Remarks

Remark 1 *The properties we have used in Theorem 1 are logically independent.*

1. *If we define for all (N, W, P), $f_i(N, W, P) = 0$ for all $i \in N$, it satisfies all the properties excepting for efficiency.*

2. *To show that null player property is independent of the rest of properties we define for all (N, W, P), $f_i(N, W, P) = \frac{1}{u}\frac{1}{|P_k|}$ for all $P_k \in P$, for all $i \in P_k$.*

3. *Symmetry inside unions is independent of the rest of the properties. For the proof we define for all (N, W, P) and for all $i \in N$*

$$f_i\left(N, W, P\right) = \begin{cases} \dfrac{i\Lambda_i(N, W, P)}{\sum_{j \in N} j\Lambda_j(N, W, P)} & \text{if } W = W_N \text{ and } P = \{\{N\}\} \\ \Lambda_i(N, W, P) & \text{otherwise} \end{cases}.$$

4. *Symmetry among unions is independent of the rest of the properties. For the proof we define for all (N, W, P) and for all $i \in N$*

$$f_i\left(N, W, P\right) = \begin{cases} \dfrac{i\Lambda_i(N, W, P)}{\sum_{j \in N} j\Lambda_j(N, W, P)} & \text{if } W = W_N \text{ or } W = \{\{1\}, \cdots, \{n\}\} \\ & \text{and } P = \{\{1\}, \cdots, \{n\}\} \\ \Lambda_i(N, W, P) & \text{otherwise} \end{cases}.$$

5. *To prove the independence of mergeability in the quotient game we define $f(N,W,P) = \Lambda(N,W,P)$ for all (N,W,P) except for the case in which $N = \{1,2,3,4,5\}$, $M^W = \{\{1,5\},\{1,2,3,4\}\}$ and $P = \{\{1\},\{2,3,4\},\{5\}\}$. For this game we define*

$$f_i(N,W,P) = \Phi_k(U,\overline{W})\frac{\Lambda_i(N,W,P)}{\sum_{j\in P_k}\Lambda_j(N,W,P)}$$

for all $P_k \in P$, for all $i \in P_k$ where Φ denotes the Shapley value.

6. *To prove the independence of mergeability inside unions we define $f(N,W,P) = \Lambda(N,W,P)$ for all (N,W,P) except for the case in which $N = \{1,2,3\}$ and $P = \{\{N\}\}$. For these games we define for all $i \in N$*

$$f_i(N,W,P) = \begin{cases} 0 & \text{if } i \text{ is a null player} \\ \frac{1}{|n(N,W)|} & \text{otherwise} \end{cases}$$

where $n(N,W)$ is the set of non-null players of the game (N,W).

7. *For all (N,W,P), we define*

$$f_i(N,W,P) = \frac{1}{\mid M^{\overline{W}} \mid}\sum_{S\in M_i^W}\frac{1}{\mid C(S)\mid}\frac{1}{\mid u(S)\mid}\frac{1}{\mid S\cap P_k\mid}$$

for all $P_k \in P$, for all $i \in P_k$ where if $S \in M^W$, $C(S) = \{T \in M^W \ / \ u(S) = u(T)\}$. This coalitional power index does not satisfy invariance with respect to essential coalitions inside unions but it satisfies the rest of the properties.

8. *To prove the independence of the property of independence of irrelevant coalitions among unions we define $f(N,W,P) = \Lambda(N,W,P)$ for all (N,W,P) except for the case in which*

$$N = \{1,2,3,4,5\},\ M^W = \{\{1,5\},\{1,2,4\},\{2,3,5\},\{1,2,5\}\} \text{ and}$$

$$P = \{\{1\},\{2,3,4\},\{5\}\}.$$

For this game we define $f_i(N,W,P) = \Lambda_i(N,W,P)$ for $i = 1,2,5$, $f_3(N,W,P) = \Lambda_3(N,W,P) - \epsilon$ and

$$f_4(N,W,P) = \Lambda_4(N,W,P) + \epsilon \text{ for some } \epsilon \approx 0.$$

Remark 2 *The definitions of the Owen value, the Banzhaf–Owen coalitional value and the symmetric coalitional Banzhaf value have interpretations that allow us look at them as a sharing of the total value in two steps. First they divide the value among the unions (by using the Shapley or the Banzhaf value of the quotient game) and in the second step they divide the value of each union within the unions (by using again the Shapley value or the Banzhaf value of a game played within the unions). We also can interpret the coalitional Deegan–Packel index as a two-step index that uses the Deegan–Packel index in both steps.*

Remark 3 *In Alonso–Meijide and Bowles (2005), new procedures based on the so-called generating functions are described to compute coalitional values for the particular case of weighted majority games. Owen (1972) proposed the multilinear extension of games as a tool to compute the Shapley value. The multilinear extension has been used to compute the Banzhaf–Owen coalitional value (Carreras and Magaña, 1994), the Owen coalitional value (Owen and Winter, 1992) and the symmetric coalitional Banzhaf value (Alonso–Meijide, Carreras and Fiestras–Janeiro, 2005). It would be of worth to study if these methods can be used to compute the coalitional Deegan–Packel power index.*

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