

# On the influence of dependent features in classification problems: a game-theoretic perspective

Laura Davila-Pena<sup>1,2</sup>, Alejandro Saavedra-Nieves<sup>3</sup>, Balbina Casas-Méndez<sup>4</sup>

<sup>1</sup>*Corresponding author. Department of Analytics, Operations and Systems, Kent Business School, University of Kent, CT2 7PE Canterbury, UK. ORCID: 0000-0003-2175-2546. [l.davila-pena@kent.ac.uk](mailto:l.davila-pena@kent.ac.uk)*

<sup>2</sup>*MODESTYA Research Group, Department of Statistics, Mathematical Analysis and Optimization, Faculty of Mathematics, University of Santiago de Compostela, Campus Vida, 15782 Santiago de Compostela, Spain.*

*[lauradavila.pena@usc.es](mailto:lauradavila.pena@usc.es)*

<sup>3</sup>*CITMAga, MODESTYA Research Group, Department of Statistics, Mathematical Analysis and Optimization, Faculty of Mathematics, University of Santiago de Compostela, Campus Vida, 15705 Santiago de Compostela, Spain. ORCID: 0000-0003-1251-6525. [alejandro.saavedra.nieves@usc.es](mailto:alejandro.saavedra.nieves@usc.es)*

<sup>4</sup>*CITMAga, MODESTYA Research Group, Department of Statistics, Mathematical Analysis and Optimization, Faculty of Mathematics, University of Santiago de Compostela, Campus Vida, 15782 Santiago de Compostela, Spain. ORCID: 0000-0002-2826-218X. [balbina.casas.mendez@usc.es](mailto:balbina.casas.mendez@usc.es)*

## Abstract

This paper deals with a new measure of the influence of each feature on the response variable in classification problems, accounting for potential dependencies among certain feature subsets. Within this framework, we consider a sample of individuals characterized by specific features, each feature encompassing a finite range of values, and classified based on a binary response variable. This measure turns out to be an influence measure explored in existing literature and related to cooperative game theory. We provide an axiomatic characterization of our proposed influence measure by tailoring properties from the cooperative game theory to our specific context. Furthermore, we demonstrate that our influence measure becomes a general characterization of the well-known Banzhaf-Owen value for games with a priori unions, from the perspective of classification problems. The definitions and results presented herein are illustrated through numerical examples and various applications, offering practical insights into our methodologies.

**Keywords:** Classification problems; Influence measure of features; Dependent features; Axiomatic characterization; Banzhaf-Owen value.

## 1 Introduction

Understanding how features influence a binary response variable becomes crucial in classification problems, with applications ranging from medical diagnosis to customer behavior

analysis. In this paper, we start from a sample of individuals for which a set of characteristics that take a finite number of values has been measured. These individuals are categorized using a classifier according to a response variable that can also take a finite set of values. Our objective is to provide a model-agnostic measure of the influence of each feature on predicting the response variable in cases where certain subsets of features are dependent. The motivation behind this research, along with its theoretical and practical significance, stems from the absence of a game theoretic-based influence measure in the literature that accounts for feature dependencies and is both experimentally validated and theoretically grounded. We fill this gap by proposing such a measure and providing an axiomatic characterization following the Banzhaf value (Banzhaf III, 1965) approach, a game theoretical solution concept closely related to the well-known Shapley value (Shapley, 1953). The main difference between the two lies in the theoretical properties they fulfill, rendering one more suitable than the other depending on the nature of the problem under consideration (Feltkamp, 1995). The Shapley value has recently found widespread application in the context of the interpretability of machine learning models (Lundberg and Lee, 2017), and both the Banzhaf and Shapley values rely on players' contributions to certain coalitions. On the other hand, the Banzhaf value has been successfully employed in various contexts to determine rankings. Recent applications include its integration into a control framework, where it offers closed-form expressions as a function of the state and establishes steady-state conditions (Muros et al., 2017). Additionally, it has been used to define a measure of political power in a Spanish autonomous community (Arévalo-Iglesias and Álvarez-Mozos, 2020) and to analyze the risk of a terrorist attack by ranking terrorists within a network (Algaba et al., 2024).

In our case, the proposed measure of feature influence emerges as a generalization of the one studied in Datta et al. (2015) for scenarios with independent features, and is identified with the Banzhaf-Owen value (G. Owen, 1981) for classification problems where both dependent features and the response variable are binary. Although the problem addressed is not inherently novel, this is the first time, to the best of our knowledge, that an influence measure grounded in the Banzhaf-Owen value is proposed, constituting the primary contribution of our work. The Banzhaf-Owen value is a cooperative game theory solution that extends upon the Banzhaf value and is particularly relevant in situations where players exhibit affiliations due to political, economic, or other considerations. In addition, we offer an axiomatic characterization of the influence measure for cases where the response variable is binary, implement it using the software R, and validate its efficacy through a variety of examples and real-world applications.

The organization of the paper is as follows. Section 2 presents a review of related literature and Section 3 offers the basic preliminaries concerning both influence measures and cooperative games. In Section 4, we provide the axiomatic characterization of our measure and Section 5 relates our proposal with the Banzhaf-Owen value. Our methodology is vali-

dated and applied in Section 6 through various numerical experiments. The paper finishes with some concluding remarks in Section 7.

## 2 Related literature

A critical challenge for the future in modern data science and artificial intelligence is achieving adequate levels of explanation for the knowledge derived from diverse methods and techniques, ensuring their appropriate utilization (Carrizosa et al., 2021). Since 2018, the European Union has recognized the so-called “right to explanation” concerning algorithms designed for decision-making processes (European Commission, 2020). Burkart and Huber (2021) discuss the reasons behind the necessity for explanations of machine learning models and the various domains where such explanations are in demand, including healthcare, the automobile industry, and recommender systems, among others. A considerable body of literature is dedicated to the pursuit and enhancement of explanation in machine learning. A prevalent objective is to quantify the influence of the characteristics on predictions, often facilitating the selection of relevant features and enabling the analysis of model behavior with a reduced set of predictor variables. Different methodologies have been explored to address the problem of explaining machine learning models. One of them involves considering permutations of the sample values (Altmann et al., 2010). Additionally, some approaches are tailored to specific methods. For instance, within the realm of classification, Ghaddar and Naoum-Sawaya (2018) propose an iterative procedure for feature selection when making use of support vector machines (SVMs). Another suggested approach consists in using concepts from cooperative game theory, assessing each feature’s contribution to prediction within any coalition of features (Carrizosa et al., 2021). Table 1 schematically shows the references cited so far, underscoring the notable interest in the literature regarding explanations for classification models and, more broadly, for machine learning.

Context	Reference	Key point
Measure of features importance	Altmann et al. (2010)	Permutations of the sample values
Feature selection	Ghaddar and Naoum-Sawaya (2018)	Classification with SVMs
Artificial intelligence	European Commission (2020)	Right of explanation
Future challenges	Carrizosa et al. (2021)	Cooperative games
Revision	Burkart and Huber (2021)	Reasons/domains

Table 1: Summary of recent references on explaining classification models.

It is also noteworthy to mention the work of Cohen et al. (2007), who present a new feature selection algorithm on the basis of Shapley’s contribution values of the features to classification precision. This method iteratively estimates the utility of features, facilitating subsequent forward selection or backward elimination procedures. Meanwhile, Štrumbelj and Kononenko (2010) provide a local measure of the influence of the features

in classification problems, i.e., operating at the individual instance level, and make use of the Shapley value. This procedure is extended in Štrumbelj and Kononenko (2011) to explain regression models and their predictions for individual instances. A paper close to the preceding two studies is Datta et al. (2015). In this work, the authors explore the global influence of different features in a classification problem. Theoretically, they restrict their influence measure to scenarios where the response variable takes only two values, and it is for these specific cases that they discuss its relation with the Banzhaf value of so-called simple cooperative games. Another relevant reference within the context of interpretability in complex machine learning models leveraging game theory is due to Lundberg and Lee (2017). They introduce SHAP (SHapley Additive exPlanations), a unified framework for interpreting predictions. This framework is supported by theoretical foundations across several prediction models, with a focus on determining the importance of the features for each specific prediction. In Casalicchio et al. (2019), alongside introducing a local measure of feature importance for individual observations and two visualization tools, the authors describe a procedure based on the Shapley value. This approach equally distributes the overall model performance among features according to their marginal contributions, enabling comparison of the importance of features across different models. Smith and Alvarez (2021) employ the methodology proposed in Lundberg and Lee (2017) to analyze a COVID-19 database collected in the early stages of the pandemic in Wuhan, China. Meanwhile, Jothi et al. (2021) concentrate on implementing a novel feature selection and data mining classifier system. According to their method, the Shapley value is set to serve as the feature selector for the data mining classifier applied to mental health data. Davila-Pena et al. (2022) expand upon the solution proposed in Štrumbelj and Kononenko (2010) by providing a global measure of the influence of features in a classification problem, along with an axiomatic characterization. The authors apply this methodology to a sample of COVID-19 patients obtained in Spain during the first wave of the pandemic, the first four-month period of 2020. Recently, Nahiduzzaman et al. (2024) introduce a novel approach for accurately classifying three types of lung cancer together with normal lung tissue by using CT (Computed Tomography) images. The integration of SHAP into their framework enhances explanatory capabilities, offering valuable insights for decision-making and bolstering confidence in real-world lung cancer diagnoses. Table 2 summarizes the references where methods derived from cooperative game theory are proposed and applied to improve the interpretation of machine learning models.

According to Štrumbelj and Kononenko (2010), the main shortcoming of existing general explanation methods is that they do not consider all possible dependencies and interactions among feature values. Since then, researchers have worked to incorporate such dependencies in their proposals, although it should be noted that dependency has not been explicitly addressed from a game theory perspective. In linear regression problems, A. B. Owen and Prieur (2017) analyze the global explanation of the model using Shap-

Context	Reference	Game theory value
Selection of features	Cohen et al. (2007)	Shapley value
Explanation of individual classifications	Štrumbelj and Kononenko (2010)	Shapley value
Explanation of regression predictions	Štrumbelj and Kononenko (2011)	Shapley value
Explanation of classification of a database	Datta et al. (2015)	Banzhaf value
Interpretation of predictions	Lundberg and Lee (2017)	Shapley value
Importance of features across models	Casalicchio et al. (2019)	Shapley value
Identifying mortality factors	Smith and Alvarez (2021)	Shapley value
Mental health data classification	Jothi et al. (2021)	Shapley value
Global explanation in classification	Davila-Pena et al. (2022)	Shapley value
Lung cancer classification	Nahiduzzaman et al. (2024)	Shapley value

Table 2: Summary of recent references on game theoretic methods for machine learning models.

ley’s contributions of the features and address the problem of dependent features using an ANOVA decomposition approach. Giudici and Raffinetti (2021) extend this framework using the so-called Lorenz Zonoid decomposition. Aas et al. (2021) address the problem of explaining individual predictions by treating dependent features using an extension of the Kernel SHAP method which is a computationally efficient approximation of Shapley values for the case of large problems. The method is illustrated with examples of linear and non-linear models.

Context	Reference	Solution concept/approach
Linear regression model	A. B. Owen and Prieur (2017)	Shapley value/ANOVA
Linear regression model	Giudici and Raffinetti (2021)	Shapley value/Lorenz Zonoids
Linear and non linear models	Aas et al. (2021)	Extended Kernel SHAP
To estimate the conditional expectations	Olsen et al. (2022)	Local Shapley value
General overview	Li et al. (2024)	Shapley/Owen values
Classification	This paper	Banzhaf-Owen value

Table 3: Summary of recent references on game theoretic methods for machine learning models with dependence among features.

In contrast to papers that use methods from the statistical domain to model dependencies between features, Olsen et al. (2022) use machine learning methods to compute local Shapley values in a regression environment. Li et al. (2024) provide a recent overview of the Shapley value as one of the main approaches from artificial intelligence to explain machine learning models, including the consideration of possible complex dependencies between features. The work reveals the potential of the Owen value (G. Owen, 1977), which is a generalization of the Shapley value suited to the case where features can be grouped into a priori coalitions. An example is given of machine learning-based decision models

in automatic driving in which the nature of traffic naturally leads to the appearance of a partitioning of features in which those associated with the vehicle would form an a priori coalition. Table 3 provides a summary of references that propose and apply methods from cooperative game theory to enhance the interpretation of machine learning models in cases where feature dependence is present.

### 3 Preliminaries

We start this section by introducing the fundamental concepts regarding datasets and influence measures necessary for the subsequent formal presentation of our model. Following this, we revisit key principles from cooperative game theory, given the close relationship between the influence measure proposed in this paper and a well-established solution concept within cooperative games.

#### 3.1 Datasets and influence measures

Let  $X = \{X_1, \dots, X_k\}$  be the set of features, with  $K = \{1, \dots, k\}$  the set of indices of the features, and  $Y$  a response variable. Also, let  $\mathcal{A}_l$  denote the finite set of possible values or states that feature  $X_l$ ,  $l \in K$ , can take,  $\mathcal{A} = \prod_{l=1}^k \mathcal{A}_l$ , and  $\mathcal{B}$  the finite set of values that variable  $Y$  can take. We will make use of datasets obtained from finite sets, denoted generally as  $N$ , consisting of  $n$  individuals. That is, we have samples, in the form of  $\mathcal{M} = \{(X^i, Y^i)\}_{i=1}^n$ , where  $X^i = (X_1^i, \dots, X_k^i)$  and  $Y^i$ ,  $i \in N$ , are the observed values of the features and the prediction of the response variable, respectively, corresponding to individual  $i$ . These predictions are obtained by a classifier trained on the same set of individuals from which the true response variable values were observed. Note that  $\cup_{i=1}^n X^i \subseteq \mathcal{A}$ , so that the set of individuals can be identified with a set of different feature profiles.

Formally, a dataset is a three-tuple  $(X, Y, \mathcal{M})$  where  $(X, Y)$  is a  $k$ -dimensional features vector and a response variable and  $\mathcal{M}$  is a sample of size  $n$ . Given a dataset  $(X, Y, \mathcal{M})$  with a binary response, i.e.,  $|\mathcal{B}| = 2$ , Datta et al. (2015) define a measure of influence of the features, for each  $l \in K$ , as follows:

$$\chi_l(X, Y, \mathcal{M}) = \sum_{(X^i, Y^i) \in \mathcal{M}} \sum_{\substack{(X_{-l}^i, a_l), b \\ a_l \in \mathcal{A}_l, b \in \mathcal{B}}} |Y^i - b|,$$

where  $(X_{-l}^i, a_l) = (X_1^i, \dots, X_{l-1}^i, a_l, X_{l+1}^i, \dots, X_k^i)$ . Note that  $\chi$  computes the number of times that a change in the state of feature  $X_l$  causes a change in the response variable.

### 3.2 Cooperative games and values

A cooperative game with transferable utility, often abbreviated as TU game, is a pair  $(G, v)$  where  $G$  denotes a finite set of players and  $v$ , the characteristic function, satisfies  $v(\emptyset) = 0$  and assigns a real number,  $v(R) \in \mathbb{R}$ , to each subset  $R \subseteq G$ . A TU game  $(G, v)$  is called a simple game if: i) it is monotonous, meaning  $v(R) \leq v(W)$  whenever  $R \subseteq W \subseteq G$ , ii)  $v(R) \in \{0, 1\}$  for all  $R \subseteq G$ , and iii)  $v(G) = 1$ . A simple game  $(G, v)$  is a weighted majority game if there exists a vector of weights  $w = (w_1, \dots, w_g)$  for the players, where  $g = |G|$ , with  $w_l \geq 0$ ,  $1 \leq l \leq g$ , and a positive real number  $a \in \mathbb{R}^+$ , referred to as the quota, such that  $v(R) = 1$ ,  $R \subseteq G$ , if and only if  $\sum_{l \in R} w_l \geq a$ .

One of the focal aspects in cooperative game theory revolves around the definition of values. Some of these values serve as procedures for distributing the worth associated to the cooperation among players, as the Shapley value and the Owen value, while others can be used as ranking indices, as the Banzhaf value and the Banzhaf-Owen value. Given a TU game  $(G, v)$  and  $l \in G$ , the Banzhaf value (Banzhaf III, 1965) is given by

$$B_l(G, v) = \sum_{R \subseteq G \setminus \{l\}} \frac{1}{2^{g-1}} \cdot (v(R \cup \{l\}) - v(R)).$$

G. Owen (1981) extends the Banzhaf value to the class of TU games with a priori unions. A TU game with a priori unions (G. Owen, 1977) is a three-tuple  $(G, v, P)$ , where  $(G, v)$  is a TU game and  $P = \{P_1, \dots, P_m\}$  is a partition of  $G$  representing affinities among players, which might stem from familiar, political, or economic motives, among other factors. We denote by  $M = \{1, \dots, m\}$  the set of indices of the coalitions, and we usually identify a coalition with its index. Given a TU game with a priori unions  $(G, v, P)$ ,  $t \in M$ , and  $l \in P_t$ , the Banzhaf-Owen value (G. Owen, 1981) is given by

$$BO_l(G, v, P) = \sum_{S \subseteq M \setminus \{t\}} \sum_{R \subseteq P_t \setminus \{l\}} \frac{1}{2^{m-1}} \cdot \frac{1}{2^{|P_t|-1}} \cdot (v(W \cup R \cup \{l\}) - v(W \cup R)),$$

where  $W = \cup_{u \in S} P_u$ . Besides, every coalition  $W \cup R$ , with  $W = \cup_{u \in S} P_u$  and  $R \subseteq P_t \setminus \{l\}$ , is compatible with partition  $P$  for  $l$  in  $P_t$ .

## 4 Main results

In this section, we will consider  $P = \{P_1, \dots, P_m\}$  a partition of  $K$  that represents possible dependencies or interactions between the values of certain subsets of features. A trivial partition is  $P^k = \{\{1\}, \dots, \{k\}\}$ , where each subset of the partition is a singleton.

Now, a dataset with interactions is a four-tuple  $(X, Y, P, \mathcal{M})$  where  $(X, Y)$  is a  $k$ -dimensional features vector and a response variable,  $P$  is a partition of  $K$ , and  $\mathcal{M}$  is a sample of size  $n$ ; in other words,  $(X, Y, \mathcal{M})$  is a dataset and  $P$  is a partition of  $K$ , the set of indices of the features.  $D(X, Y)$  denotes the family of all datasets with interactions where  $(X, Y)$

are the features vector and the response variable. Our goal is to make use of techniques used in classification problems to define a measure for studying the influence of features on the predicted value of the response variable under the assumption of possible dependencies between subsets of these features. First, let us state the formal definition of such an influence measure within this context.

**Definition 4.1.** An influence measure for  $D(X, Y)$  is a map,  $I$ , that assigns to every dataset with interactions,  $(X, Y, P, \mathcal{M}) \in D(X, Y)$ , a vector  $I(X, Y, P, \mathcal{M}) \in \mathbb{R}^k$ . The measure  $I_l(X, Y, P, \mathcal{M}) \in \mathbb{R}$ ,  $l \in K$ , is a metric of the importance of  $X_l$  in determining the predicted values of  $Y$  over  $\{(X^i)\}_{i=1}^n$ .

The main objective is to show that there is a unique measure of influence that satisfies a set of natural axioms, which we introduce and describe below. In what follows, we assume that  $|\mathcal{B}| = 2$ , i.e., the response variable  $Y$  can only take two distinct values.

A feature  $X_l$ , with  $l \in K$ , is said to be *non-influential* in the dataset with interactions  $(X, Y, P, \mathcal{M})$  if  $Y^i = Y^j$  for all  $i, j \in N$  such that  $X_{-l}^i = X_{-l}^j$ , where  $X_{-l}^i$ , with  $i \in N$  and  $l \in K$ , denotes the vector  $X^i$  after removing the  $l$ -th coordinate. First, we introduce the dummy property for influence measures.

**(DP) Dummy property.** An influence measure  $I$  satisfies the *dummy property* if, for every  $(X, Y, P, \mathcal{M}) \in D(X, Y)$  and every non-influential feature in the dataset  $(X, Y, P, \mathcal{M})$ ,  $X_l$ , with  $l \in K$ , it holds that  $I_l(X, Y, P, \mathcal{M}) = 0$ .

Given a dataset with interactions  $(X, Y, P, \mathcal{M})$  and a bijective mapping  $\sigma$  from  $K$  to itself, we define  $\sigma(X, Y, P, \mathcal{M}) = (\sigma(X), Y, \sigma(P), \sigma(\mathcal{M}))$  in the natural way, consisting in relabelling the features according to  $\sigma$ , i.e., making the index of  $l$ , with  $l \in K$ , in the initial dataset become now  $\sigma(l)$ . We write  $\sigma(\mathcal{M}) = \{(\sigma(X^i), Y^i)\}_{i=1}^n$ . Given a bijective mapping  $\tau$  from  $\mathcal{A}_l$ ,  $l \in K$ , to itself, we define  $\tau(X, Y, P, \mathcal{M}) = (X, Y, P, \tau(\mathcal{M}))$  in a similar manner, consisting of relabelling the values of  $\mathcal{A}_l$  according to  $\tau$ , i.e., making the value of  $a_l$ , with  $a_l \in \mathcal{A}_l$ , in the initial dataset becoming now  $\tau(a_l)$ . We write  $\tau(\mathcal{M}) = \{(\tau(X^i), Y^i)\}_{i=1}^n$ .

These concepts enable us to introduce various notions of symmetry that measures of influence, like those examined here, should satisfy.

**(FSY) Feature symmetry.** An influence measure  $I$  satisfies *feature symmetry property* if, for every  $(X, Y, P, \mathcal{M}) \in D(X, Y)$  such that  $P = P^k$  and a bijective mapping  $\sigma$  from  $K$  to itself, it holds that  $I_l(X, Y, P, \mathcal{M}) = I_{\sigma(l)}(\sigma(X, Y, P, \mathcal{M}))$  for all  $l \in K$ .

**(SSY) State symmetry.** An influence measure  $I$  satisfies *state symmetry property* if, for every  $(X, Y, P, \mathcal{M}) \in D(X, Y)$  such that  $P = P^k$  and a bijective mapping  $\tau$  from  $\mathcal{A}_l$ ,  $l \in K$ , to itself, it holds that  $I_q(X, Y, P, \mathcal{M}) = I_q(\tau(X, Y, P, \mathcal{M}))$  for all  $q \in K$ .

**(SY) Symmetry.** An influence measure  $I$  satisfies *symmetry property* if it satisfies both feature symmetry (FSY) and state symmetry (SSY).

Below we present some common properties that an influence measure should satisfy when considering a partition structure. Since  $|\mathcal{B}| = 2$ , we can assume that  $\mathcal{B} = \{0, 1\}$ . Consequently, we define  $W(\mathcal{M}) = \{i \in N : Y^i = 1\}$  and  $L(\mathcal{M}) = \{i \in N : Y^i = 0\}$  as the sets of sample profiles where the response variable takes the values 1 and 0, respectively. Thus, in general, given a set of individuals  $N$ , with  $n \leq |\mathcal{A}|$ ,  $W, L \subseteq N$ , and  $W \cap L = \emptyset$ , we can identify a sample with  $(W, L)$  and a dataset with interactions with  $(X, Y, P, (W, L))$ .

**(DU) Disjoint union.** An influence measure  $I$  satisfies *disjoint union property* if for every  $(X, Y, P, (Q, R \cup R')), (X, Y, P, (R \cup R', Q)) \in D(X, Y)$  where  $X$  is a set of features and  $Q, R$ , and  $R'$  are pairwise disjoint sets satisfying  $|Q \cup R \cup R'| \leq |\mathcal{A}|$  for all  $l \in K$ , it holds that

$$I_l(X, Y, P, (Q, R)) + I_l(X, Y, P, (Q, R')) = I_l(X, Y, P, (Q, R \cup R'))$$

and

$$I_l(X, Y, P, (R, Q)) + I_l(X, Y, P, (R', Q)) = I_l(X, Y, P, (R \cup R', Q)).$$

**(II) Indifference to interactions.** An influence measure  $I$  satisfies *indifference to interactions property* if for all  $(X, Y, P, \mathcal{M}) \in D(X, Y)$ , for all  $t \in M$ , with  $l, q \in P_t$  and  $l \neq q$ , it holds that  $I_l(X, Y, P, \mathcal{M}) = I_l(X, Y, P_{-q}, \mathcal{M})$ , where  $P_{-q}$  is the partition that results from removing  $q$  from its original subset to create a unitary subset, i.e.,  $P_{-q} = \{P_1, \dots, P_{t-1}, P_t \setminus \{q\}, P_{t+1}, \dots, P_m, \{q\}\}$ .

**(RP) Relevance of dependent feature profiles.** An influence measure  $I$  satisfies *relevance of dependent feature profiles property* if for all  $(X, Y, P, \mathcal{M}) \in D(X, Y)$  and for all  $t \in M$  such that  $|P_t| = 1$ , with  $l \in P_t$ , it holds that  $I_l(X, Y, P, \mathcal{M}) = I_l(X, Y, P^k, \mathcal{M}^t)$ , where

$$\mathcal{M}^t = \{(X^i, Y^i) \in \mathcal{M}, i \in N : \text{if } u \in M \setminus \{t\}, \text{ then } X_q^i \approx X_v^i \text{ for all } q, v \in P_u\} \quad (1)$$

is the subsample of  $\mathcal{M}$  where features within the same union, except for  $P_t$ , precisely adhere to a pre-adjusted dependency model. In a union consisting of two binary features, the acceptable values for both features will be either identical or opposite, depending on whether the dependency is positive or negative.

Note that each of these properties extends specific axioms from the cooperative game theory literature to the context of influence measures. For instance, (DP) is an extension of the null player property utilized in the axiomatization of solutions for TU games, as evidenced in works such as Feltkamp (1995) for the Banzhaf value. Datta et al. (2015) also employ this property from a binary classification perspective. Additionally, (FSY), (SSY), and (SY) share similarities with properties outlined in Datta et al. (2015). These properties, in turn, expand upon those used in the axiomatization of the Banzhaf-Owen value for a TU game with a priori unions, among others, as seen in works like Alonso-Meijide et al.

(2007). Furthermore, (DU) has the same essence as the union-intersection axiom employed in Lehrer (1988) to characterize the Banzhaf value for TU games. It also serves as a generalization of the axiom with the same name used in Datta et al. (2015). Lastly, (II) and (RP) extend properties of indifference in unions and the quotient game for single-player unions introduced in Alonso-Mejide et al. (2007) to axiomatize the Banzhaf-Owen value.

The following proposition extends the result of Datta et al. (2015) to the case of a set of categorical responses, when there is no partition structure on the affinities of the features.

**Proposition 4.2.** *An influence measure for  $D(X, Y)$ ,  $I$ , satisfies (DP), (SY), and (DU) if and only if there exists a constant  $C$  such that for every dataset with interactions  $(X, Y, P^k, \mathcal{M})$  and every feature  $l \in K$ ,*

$$I_l(X, Y, P^k, \mathcal{M}) = C \cdot \sum_{(X^i, Y^i) \in \mathcal{M}} \sum_{\substack{((X_{-l}^i, a_l), b) \in \mathcal{M}: \\ a_l \in \mathcal{A}_l, b \in \mathcal{B}}} |Y^i - b|. \quad (2)$$

Moreover, it holds that  $I_l(X, Y, P^k, \mathcal{M}) = C \cdot \chi_l(X, Y, \mathcal{M})$ .

Now, we present an axiomatic characterization of our influence measure in cases where a partition exists over the set of features, reflecting their dependencies and interactions.

*Remark 4.1.* It is important to note the analogy of this result with that obtained by Alonso-Mejide et al. (2007) for the Banzhaf-Owen value in the context of TU games with a priori unions.

**Theorem 4.3** (Existence and uniqueness). *An influence measure for  $D(X, Y)$ ,  $I$ , satisfies (DP), (SY), (DU), (II), and (RP) if and only if there exists a constant  $C$  such that for every dataset with interactions  $(X, Y, P, \mathcal{M})$ ,  $t \in M$ ,  $l \in P_t$ , it holds that*

$$I_l(X, Y, P, \mathcal{M}) = C \cdot \Psi_l(X, Y, P, \mathcal{M}), \quad (3)$$

where

$$\Psi_l(X, Y, P, \mathcal{M}) = \sum_{(X^i, Y^i) \in \mathcal{M}^t} \sum_{\substack{((X_{-l}^i, a_l), b) \in \mathcal{M}^t: \\ a_l \in \mathcal{A}_l, b \in \mathcal{B}}} |Y^i - b|.$$

## 5 The influence measure and games with a priori unions

In this section, we study the proposed measure of influence from a game-theoretical perspective. For this purpose, we mainly follow the ideas in Datta et al. (2015).

First, we will consider a TU game associated to any sample. Let  $\mathcal{M} = \{(X^i, Y^i)\}_{i=1}^n$  be a sample such that  $\mathcal{A} = \cup_{i=1}^n X^i$  and  $|\mathcal{A}_l| = 2$ , for all  $l \in K$ . Thus, the sample  $\mathcal{M}$  corresponds to the TU game  $(K, v^{\mathcal{M}})$  defined, for all  $R \subseteq K$ , by

$$v^{\mathcal{M}}(R) = Y^i \iff \exists i \in N \text{ such that } X_l^i = \begin{cases} 1 & \text{if } l \in R, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

If, moreover, we particularly consider the case where  $|\mathcal{B}| = 2$ , we can assume that the

characteristic function of the game fulfills  $v^{\mathcal{M}}(R) \in \{0, 1\}$ , for all  $R \subseteq K$ .

Using this proposal of TU game, Datta et al. (2015) innovatively relate their introduced influence measure to the Banzhaf value of the TU game  $(K, v^{\mathcal{M}})$ . More precisely, when the sample  $\mathcal{M}$  corresponds to the TU game  $(K, v^{\mathcal{M}})$  and  $|\mathcal{B}| = 2$ , it follows that

$$B(K, v^{\mathcal{M}}) = \frac{\chi(X, Y, \mathcal{M})}{|\mathcal{A}|}, \quad (5)$$

indicating that the influence measure coincides with the (raw) Banzhaf value.

However, as previously justified, it is natural to assume the presence of a partition of the features describing potential dependencies among them. Consequently, a dataset with interactions  $(X, Y, P, \mathcal{M})$ , with  $\mathcal{M}$  corresponding to a TU game  $(K, v^{\mathcal{M}})$  and  $P$  a partition of  $K$ , can be identified with a TU game with a priori unions  $(K, v^{\mathcal{M}}, P)$ .

The following result extends Equation (5) to the case of dependent features and shows that the influence measure for potential dependent features introduced in this paper (see Equation (3)) is a generalization of the Banzhaf-Owen value for simple games with a priori unions.

**Proposition 5.1.** *Let  $(X, Y, P, \mathcal{M})$  be a dataset with interactions where the associations between the features within each union are positive. Suppose that  $\mathcal{A} = \cup_{i=1}^n X^i$ ,  $|\mathcal{A}_l| = 2$  for all  $l \in K$ , and  $|\mathcal{B}| = 2$ . If  $(K, v^{\mathcal{M}})$  is the TU game corresponding to  $\mathcal{M}$ , then it holds that, for every  $t \in M$ ,  $l \in P_t$ ,*

$$BO_l(K, v^{\mathcal{M}}, P) = \frac{\Psi_l(X, Y, P, \mathcal{M})}{|\mathcal{A}^t|}, \quad (6)$$

where  $\mathcal{A}^t = \{a = (a_1, \dots, a_k) \in \mathcal{A} : \text{if } u \in M \setminus \{t\} \text{ then } a_q = a_v \text{ for all } q, v \in P_u\}$ .

Below, we consider an example that illustrates the previous result.

**Example 5.2.** *Let us take the dataset with interactions  $(X, Y, P, \mathcal{M})$  where  $X = \{X_1, X_2, X_3, X_4\}$ , that is,  $k = 4$ , and such that  $\mathcal{A}_l = \mathcal{B} = \{0, 1\}$  for all  $l \in K$ . Suppose that  $\mathcal{A} = \cup_{i=1}^n X^i$  and  $n = 16$ . In addition, for  $i \in N$ , we have that*

$$Y^i = \begin{cases} 1 & \text{if } X^i \in \{0011, 1110, 1011, 0111, 1111\}, \\ 0 & \text{otherwise.} \end{cases}$$

The dataset with interactions  $(X, Y, P, \mathcal{M})$  can be identified with a weighted majority game with a priori unions  $(K, v^{\mathcal{M}}, P)$  where, for example, the vector of weights is  $w = (1, 1, 4, 3)$  and the quota is  $a = 6$ . Table 4 displays the influence measure introduced in this paper for the above-specified dataset, considering all possible partitions of the feature set. By Proposition 5.1, this influence measure coincides with the Banzhaf-Owen value. All results have been computed using the R library `powerindexR`<sup>1</sup>.

From Table 4, we can clearly observe the behavior of various properties of our influence measure.

---

<sup>1</sup>More information is available at <https://cran.r-project.org/web/packages/powerindexR/index.html>.

Scenario	Feature set partitions ( $P$ )	Influence measures of the features ( $BO$ )
1	$\{\{1\}, \{2\}, \{3\}, \{4\}\}$	(0.125, 0.125, 0.625, 0.375)
2	$\{\{1\}, \{2\}, \{3, 4\}\}$	(0.000, 0.000, 0.625, 0.375)
3	$\{\{1\}, \{3\}, \{2, 4\}\}$	(0.000, 0.125, 0.500, 0.375)
4	$\{\{1\}, \{4\}, \{2, 3\}\}$	(0.250, 0.125, 0.625, 0.250)
5	$\{\{2\}, \{3\}, \{1, 4\}\}$	(0.125, 0.000, 0.500, 0.375)
6	$\{\{2\}, \{4\}, \{1, 3\}\}$	(0.125, 0.250, 0.625, 0.250)
7	$\{\{3\}, \{4\}, \{1, 2\}\}$	(0.125, 0.125, 0.750, 0.250)
8	$\{\{1, 2\}, \{3, 4\}\}$	(0.000, 0.000, 0.750, 0.250)
9	$\{\{1, 3\}, \{2, 4\}\}$	(0.000, 0.250, 0.500, 0.250)
10	$\{\{1, 4\}, \{2, 3\}\}$	(0.250, 0.000, 0.500, 0.250)
11	$\{\{1\}, \{2, 3, 4\}\}$	(0.000, 0.125, 0.625, 0.375)
12	$\{\{2\}, \{1, 3, 4\}\}$	(0.125, 0.000, 0.625, 0.375)
13	$\{\{3\}, \{1, 2, 4\}\}$	(0.125, 0.125, 0.500, 0.375)
14	$\{\{4\}, \{1, 2, 3\}\}$	(0.125, 0.125, 0.625, 0.000)
15	$\{\{1, 2, 3, 4\}\}$	(0.125, 0.125, 0.625, 0.375)

Table 4: Numerical results for our influence measure in the 15 possible scenarios of Example 5.2.

Notably, in scenarios 1 and 2, features 3 and 4 exhibit identical influence, reflecting the property of indifference to interactions. This property is further confirmed by looking at the influence of features 3 and 4 in scenarios 2 and 12. Additionally, in scenario 1, where all feature unions have size 1, every feature is influential. However, when dependencies between groups of two or three features are considered, some features become non-influential, as demonstrated in scenarios 9 and 10.  $\triangle$

In general, obtaining the exact Banzhaf values entails high computational complexity, specially for large sets of players, as the number of elements to be evaluated increases exponentially (cf. Deng and Papadimitriou, 1994). While there are procedures to compute Banzhaf values for certain classes of games, which mitigate this issue, our approach does not benefit from such optimizations. Our influence measure requires the evaluation of all elements in the considered sample  $\mathcal{M}$ .

## 6 Numerical results

This section analyzes the performance of our influence measure on two different datasets that will be further described below. The proposed influence measure has been implemented in R 4.3.3 and a set of numerical experiments were run on a quad-core Intel i7-8665U CPU with 16 GB RAM.

We compute our influence measure for each dataset in several scenarios. On the one hand, we consider three different predictive models from the families of random forests (RFs), support vector machines (SVMs), and logistic regression (LR), chosen for their well-documented performance as classifier types (Fernández-Delgado et al., 2014). We use the base implementations of these models provided by the RWeka<sup>2</sup> library in R software, specifically the Breiman’s random forest classifier (Breiman, 2001), the Platt’s sequential minimal optimization (SMO) algorithm for training a support vector machine classifier (Platt, 1998),

<sup>2</sup>More information is available at <https://cran.r-project.org/web/packages/RWeka/RWeka.pdf>.

and a modified le Cessie and van Houwelingen’s multinomial logistic regression model with a ridge estimator (le Cessie and van Houwelingen, 1992), respectively. On the other hand, we consider three distinct coalitional structures: 1) a partition formed by singletons, 2) a manually selected partition, and 3) a partition based on hierarchical clustering. For this latter case, the number of clusters is selected to match the number of coalitions in the manual partition, and we use the Jaccard distance measure, given the binary nature of our datasets.

The following subsections report the computational study. Subsection 6.1 explores the influence of certain factors on the severity of a car crash, while Subsection 6.2 examines the influence of listening to various music groups or singers on the likelihood of listening to other artists. We will also discuss how the chosen classifiers and partitions impact feature rankings. All the datasets used are available at [https://github.com/LauraDavilaPena/GT-based\\_IM](https://github.com/LauraDavilaPena/GT-based_IM).

## 6.1 On the analysis of car crash fatalities

In this section, we apply our proposed influence measure to the relevant context of occupant safety in car crashes. The severity of an accident can vary significantly based on the type of collision. For instance, a head-on collision differs substantially from a side-impact collision or a vehicle rollover. Furthermore, the risk of fatality in a car older than ten years old is twice as high as in a newer vehicle. We will specifically look at the variables that influence the likelihood of fatalities in vehicle accidents.

To achieve this, we examine the influence of various factors that help us describe the nature of a road accident. Specifically, we consider the variables included in the `nassCDS` dataset from R software. This dataset contains information on car crashes in the US in the period 1997-2002, reported by the police, in which there is an injury (to person or property) and at least one vehicle is towed. The data is limited to front-seat occupants and includes only a subset of the recorded variables, with additional restrictions. For our application, we have conveniently adapted this database to focus on binary variables. The transformed database consists of a sample of 17,565 observations for the 10 characteristics listed in Table 5.

From the sample, we obtain the influence measure both with and without a coalitional structure of the features across various scenarios. Initially, we consider the case of partitioning with unitary elements, meaning we do not account for potential affinity relations between the features; this corresponds to the original case presented by Datta et al. (2015). Next, we examine a constructed-by-design partition (CDP) where features 1, 4, and 9 are grouped in one block, 2, 6, and 8 in another block, and features 3, 5, 7, and 10 act individually. Finally, we adopt a coalitional structure specified by hierarchical clustering, fixing the number of clusters at six. This hierarchical clustering partition (HCP) joins features 2, 7, and 9 in the same block, features 4, 5, and 8 in another block, and leaves 1, 3, 6, and

Response	Description
deceased	Binary variable indicating whether the person involved in the car accident is deceased (1) or not (0).
Feature	Description
1 dvcat	Binary variable indicating whether the vehicle, at the moment of the accident, was traveling at a speed higher than 55 km/h (1) or not (0).
2 airbag	Binary variable indicating whether the vehicle had an airbag system (1) or not (0).
3 seatbelt	Binary variable indicating whether the person involved was wearing a seat belt (1) or not (0).
4 frontal	Binary variable indicating whether the vehicle crash was frontal (1) or non-frontal (0).
5 sex	Binary variable indicating the sex of the person involved: 1 for male and 0 for female.
6 abcat	Binary variable indicating airbag activation: 1 if one or more airbags in the vehicle were activated (even if not deployed) and 0 if none were deployed (either due to malfunction or being disabled).
7 occRole	Binary variable indicating whether the person involved was the driver (1) or a passenger (0) of the vehicle.
8 deploy	Binary variable indicating whether the airbag functioned correctly (1) or was unavailable or not functioning (0).
9 ageOFocc	Binary variable indicating whether the person was 30 years old or less (1) or over 30 years old (0).
10 age	Binary variable indicating whether the vehicle was 10 years old or more (1) or less than 10 years old (0).

Table 5: Summary of the considered features in the analysis of car crash fatalities.

10 alone. For each scenario, we compute the resulting influence measure (IM) using the above-specified random forest (RF), support vector machine (SVM), and logistic regression (LR) classifiers as predictive models. Table 6 displays the numerical results.

Feature	Datta et al.'s IM, (2)			IM with CDP, (3)			IM with HCP, (3)		
	RF	SVM	LR	RF	SVM	LR	RF	SVM	LR
1	0.42801	0	0.19516	0.63830	0	0.53216	0.03189	0	0.03826
2	0.00865	0	0.00159	0	0	0	0.00798	0	0.00059
3	0.02847	0	0.08289	0	0	0	0.01934	0	0.03491
4	0.11967	0	0.34728	0.15016	0	0.27097	0.13540	0	0.13117
5	0.03450	0	0.01697	0	0	0	0.00106	0	0
6	0.03507	0	0.00649	0.01045	0	0	0.00319	0	0
7	0	0	0	0	0	0	0	0	0
8	0.03632	0	0.00330	0.03947	0	0	0.02977	0	0
9	0.02812	0	0.004787	0.05136	0	0.01663	0.02157	0	0.00177
10	0.01754	0	0.00137	0	0	0	0	0	0

Table 6: Numerical results in the analysis of car crash fatalities.

Based on these findings, several conclusions can be drawn. Features 1 and 4 consistently hold the top two positions in the ranking of the most influential features. Specifically, they refer to the vehicle's speed and whether the collision is frontal. Factors such as airbag functioning (feature 8), seat belt usage (feature 3), or the person's age (feature 9) rank third

in the different rankings obtained, as can be seen in Table 7. Notably, regardless of the scenario examined, the individual’s role (passenger or driver), represented by feature 7, appears to have no influence.

Position	Datta et al.’s IM, (2)			IM with CDP, (3)			IM with HCP, (3)		
	RF	SVM	LR	RF	SVM	LR	RF	SVM	(LR)
1	1	-	4	1	-	1	4	-	4
2	4	-	1	4	-	4	1	-	1
3	8	-	3	9	-	9	8	-	3

Table 7: Top three features in the analysis of car crash fatalities.

It is also worth noting that the SVM classifier does not yield conclusive results when using measures based on Datta et al.’s methodology. In such cases, influence measures always return a value of 0. As discussed in the literature, the imbalanced distribution between 0s and 1s in the response variable can lead to influence measure outcomes that may not accurately reflect reality. As shown in Table 8, SVM does not predict any occurrences of the value 1 for the response variable in this particular case study.

Value	Original response	RF	SVM	LR
0	16,788	17,496	17,565	17,464
1	777	69	0	101

Table 8: Summary of the responses from both the original database and those predicted by the classifiers.

To check this conjecture, we draw a subsample from the original database consisting of 777 instances with a response value of 1 and 777 instances with a response value of 0 (balanced in response). Table 9 presents the numerical results for this ad hoc subsample. Interestingly, in this case, we observe that the influence measures based on Datta et al.’s methodology are not zero for SVM’s scenarios.

Feature	Datta et al.’s IM, (2)			IM with CDP, (3)			IM with HCP, (3)		
	RF	SVM	LR	RF	SVM	LR	RF	SVM	LR
1	1.94852	2.01931	2.23681	2.70455	2.70455	4.04546	4.09669	4.09669	4.95165
2	0.22780	0.17503	1.24324	0.39351	0.49087	3.61460	0.34128	0.02202	0.45505
3	8.15187	3.60489	7.11454	6.87500	1.50000	0	15.96947	6.52417	14.97201
4	3.33333	6.39768	5.10296	2.23864	1.97159	1.86932	4.59246	8.93836	7.13340
5	0.79022	1.10039	0	1.64286	0	0	0.48574	1.57314	0
6	3.78765	2.50708	4.77992	1.08316	3.22110	3.78093	9.48092	4.04580	10.22392
7	0	0	0	0	0	0	0	0	0
8	1.57143	2.04376	0	3.33874	3.39148	0	1.19411	1.67065	0
9	0.00257	0	0.01030	0.01136	0	0.04546	0	0	0.02202
10	0.71429	0.24710	0	0.19643	0	0	1.52163	0	0

Table 9: Numerical results in the analysis of car crash fatalities over the considered subsample.

A consistent finding shared with the original case is the lack of influence of feature 7 (the passenger’s role) across all studied scenarios. Furthermore, Table 10 presents additional insights, showcasing the top three features.

Features 3, 4, and 6, which relate to seat belt usage, crash type, and airbag activation,

Position	Datta et al.'s IM, (2)			IM with CDP, (3)			IM with HCP, (3)		
	RF	SVM	LR	RF	SVM	LR	RF	SVM	LR
1	3	4	3	3	8	1	3	4	3
2	6	3	4	8	1	6	6	3	6
3	4	6	6	1	6	2	4	1	4

Table 10: Top three features in the analysis of car crash fatalities over the considered subsample.

respectively, are the most influential factors (with slight variations in order) for both the influence measure without coalitional structure or with coalitional structure provided by hierarchical clustering. It is reasonable to assume that safety features such as seat belts and airbags play a pivotal role in ensuring passenger safety. In the hierarchical clustering scenario with SVM, feature 6 is replaced by feature 1. More disparities are found when considering the partition constructed by mere feature observation. Only feature 1 ranks in the top three across all scenarios. Feature 8 emerges when RF and SVM are selected, while feature 6 appears alongside SVM and LR. Feature 3 completes the ranking given by RF, and feature 2 does so for LR.

As a final thought, the numerical results presented here seem to be strongly dependent on the selected sample and the methodology employed in each case. Nevertheless, it is notable that the ultimate discrepancies are minimal in terms of rankings, with only a few variations in the positions. The incorporation of information regarding feature affinities through partitions may justify this fact.

## 6.2 On the analysis of musical taste in Spotify

Next, we use our proposed influence measure to analyze musical tastes from a Spotify database. Our goal is to identify the most influential singers or music groups. The original dataset contains a total of 285 artists and 1226 users and includes which users have listened to which artists. For the purposes of our study, we have decided to consider only the 75 most listened-to artists, all of them with an audience share of no less than 5%, to ensure results in a reasonable computational time and to avoid examining those artists who are already of little interest.

Table 11 presents the selected artists, showcasing the percentage of listeners and the decade in which their band was created. This will constitute the basis for our CDP, which groups artists based on the decade of their foundation. Table 12 summarizes the number of artists in each group.

To apply our methodology, we first need to define our classification problems. We take 10 of these artists as the response variable for each of our problems, and our objective will be to study whether or not listening to the other 74 artists affects this target. In this example, we compare the results obtained in two different scenarios. First, we consider the case where the partition is determined by the decade of the band's foundation, the CDP. Then, as in Subsection 6.1, we consider hierarchical clustering techniques to obtain a partition

Artist	Decade	% of listeners	Artist	Decade	% of listeners
linkin.park	1990	16.2	johnny.cash	1950	7.3
coldplay	1990	16.1	kings.of.leon	2000	7.3
red.hot.chili.peppers	1980	15.1	amy.winehouse	2000	7.2
rammstein	1990	15	depeche.mode	1980	7.2
system.of.a.down	1990	13.2	bullet.for.my.valentine	1990	7
metallica	1980	12.2	in.extremo	1990	7
die.toten.hosen	1980	11.8	blink.182	1990	6.9
billy.talent	2000	10.8	slipknot	1990	6.7
the.killers	2000	10.8	death.cab.for.cutie	1990	6.5
the.beatles	1960	10.6	daft.punk	1990	6.4
jack.johnson	2000	10	limp.bizkit	1990	6.3
muse	1990	10	sum.41	2000	6.2
beatsteaks	1990	9.5	fall.out.boy	2000	6.1
foo.fighters	1990	9.5	schandmaul	1990	6.1
nirvana	1980	9.5	kanye.west	2000	6
radiohead	1990	9.4	seeed	1990	6
arctic.monkeys	2000	9.1	tenacious.d	1990	6
placebo	1990	9.1	rihanna	2000	6
bloc.party	2000	8.8	papa.roach	1990	5.8
evanescence	1990	8.8	portishead	1990	5.8
rise.against	2000	8.7	rage	1990	5.8
the.kooks	2000	8.7	franz.ferdinand	2000	5.7
mando.diao	2000	8.6	marilyn.manson	1980	5.6
the.white.stripes	1990	8.4	nelly.furtado	2000	5.6
deichkind	1990	8.3	queen	1970	5.5
incubus	1990	8.3	bob.marley	1960	5.5
farin.urlaub	1990	8.2	feist	1990	5.5
in.flames	1990	8.2	massive.attack	1980	5.4
clueso	2000	8.1	queens.of.the.stone.age	1990	5.4
peter.fox	2000	8	iron.maiden	1970	5.2
the.offspring	1980	8	avril.lavigne	2000	5.1
air	1990	7.8	amon.amarth	1990	5.1
subway.to.sally	1990	7.7	apocalyptica	1990	5.1
nightwish	1990	7.7	gorillaz	1990	5.1
the.prodigy	1990	7.4	nine.inch.nails	1980	5.1
disturbed	1990	7.3	oasis	1990	5.1
ac.dc	1970	7.3	children.of.bodom	1990	5
green.day	1980	7.3			

Table 11: List of the 75 most listened-to music bands in the database, with the prescribed partition by decade of the band’s creation and percentage of users who listen to it.

Decade	1950	1960	1970	1980	1990	2000
# of artists	1	2	3	10	40	19

Table 12: Distribution of artists per decade.

of the bands for each of the considered response variables, that is, the HCP. Note that the large number of bands involved makes it computationally infeasible to calculate exactly the influence measure when the partition is composed of singletons, that is, Datta et al.’s influence measure (2).

Table 13 summarizes the three most influential bands for each scenario. The numerical results show some similarities, which are detailed below. For instance, according to RF, Avril Lavigne, Muse, and Air are the three most influential bands for listening to Coldplay. In the case of The Killers, Green Day is the most influential band; for The Beatles, Kings of Leon; for Muse, Avril Lavigne and Evanescence swap positions when using the CDP

		IM with CDP, (3)			IM with HCP, (3)		
		Position 1	Position 2	Position 3	Position 1	Position 2	Position 3
coldplay	RF	avril.lavigne	muse	air	avril.lavigne	muse	air
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
metallica	RF	nightwish	rage	evanescence	nightwish	iron.maiden	amon.amarth
	SVM	-	-	-	-	-	-
	LR	-	-	-	nightwish	iron.maiden	-
the.killers	RF	green.day	the.white.stripes	kings.of.leon	green.day	franz.ferdinand	evanescence
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
the.beatles	RF	kings.of.leon	air	franz.ferdinand	kings.of.leon	muse	apocalyptica
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
muse	RF	avril.lavigne	evanescence	placebo	evanescence	avril.lavigne	-
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
ac.dc	RF	rage	air	the.prodigy	rage	evanescence	air
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
amy.winehouse	RF	kings.of.leon	nelly.furtado	peter.fox	kings.of.leon	bob.marley	the.beatles
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
rihanna	RF	limp.bizkit	the.prodigy	amy.winehouse	evanescence	seeed	nelly.furtado
	SVM	nelly.furtado	kanye.west	-	seeed	nelly.furtado	kanye.west
	LR	nelly.furtado	kanye.west	-	seeed	nelly.furtado	kanye.west
queen	RF	amon.amarth	the.prodigy	muse	feist	muse	the.beatles
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-
bob.marley	RF	seeed	jack.johnson	the.killers	seeed	the.killers	peter.fox
	SVM	-	-	-	-	-	-
	LR	-	-	-	-	-	-

Table 13: Top three bands in the analysis of musical taste in Spotify.

and the HCP; for AC/DC, Rage and Air are both in the top three; for Amy Winehouse, the most influential band is Kings of Leon; and for Queen, the only coincidence is that Muse is in position 3 when using the CDP and in position 2 when using the HCP. Finally, Seeed is the most influential band for listening to Bob Marley, although The Killers are in positions 3 and 2 for the CDP and the HCP, respectively. In all these cases, no other bands are influential for SVM and LR. However, for Metallica, Nightwish is the most influential band according to RF and both partition structures, as well as LR with the HCP. Also in this latter case, Iron Maiden is in the second position under RF and LR. When analyzing the case of Rihanna, Nelly Furtado ranks first for the CDP with SVM and LR, second for the HCP with SVM or LR, and third for the HCP with RF. Seeed is the most influential when using the HCP with SVM and LR, and ranks second with RF. We also highlight the case of Kanye West, in positions 2 and 3 for both SVM and LR with CDP and HCP, respectively.

### 6.2.1 The case of the 15 most listened-to music bands

As the reader can check in the previous section, the SVM and LR classifiers also did not yield conclusive results in the analysis of musical tastes. Therefore, we have narrowed our study to focus only on the 15 most listened-to bands in our original dataset. According to Table 11, these bands have been listened to by more than 9.5% of the users included in the database.

The outline of this section follows our previous approach. We once again apply RF, SVM, and LR as classifiers on the dataset under consideration, and we obtain the influence measure using the two aforementioned partitions: CDP and HCP. Despite focusing on the

15 most listened-to bands, we encountered inconclusive results with SVM when assessing influences.

First, we analyze the case of using the CDP. The corresponding numerical results are shown in Table 14. For each classification problem (by columns), we highlight the most influential band in bold. Using RF, the following conclusions can be drawn. Linkin Park is the most influential band for listening to System of a Down, Metallica and Die Toten Hosen; Rammstein, to Billy Talent; System of a Down, to Rammstein; Die Toten Hosen, to Nirvana; Billy Talent, to Linkin Park and Muse; Muse, to Beatsteaks; Beatsteaks, to Red Hot Chili Peppers, The Killers, and Jack Johnson; Foo Fighters, to Coldplay; and Nirvana, to The Beatles. When considering LR, Coldplay and Foo Fighters are both the most influential bands on Beatsteaks; Rammstein, on Metallica and Die Toten Hosen; Metallica, on The Beatles; Muse, on Rammstein; Beatsteaks, on The Killers and Foo Fighters; and Foo Fighters, on Coldplay, Billy Talent, Muse, and Beatsteaks. Only Foo Fighters and Beatsteaks share a bilateral relation where each band is mutually the most influential on the other.

In view of Table 14, the most influential bands do not seem to coincide in the rankings resulting from using RF and LR as classifiers. However, we will use Pearson correlations on the numerical results as a measure of overall comparison of the rankings obtained (see Table 15). The greatest similarities between rankings are found, in this order, when we study the influence of listening to The Killers, Coldplay, Foo Fighters, The Beatles and Die Toten Hosen. All of these bands have a correlation of around or above 0.6.

Second, we do a similar study from the results obtained with the partitions prescribed by the hierarchical clustering. In this case, using RF as a classifier, Red Hot Chili Peppers is the most influential on listening to Coldplay and Metallica according to our influence measure; Rammstein, on System of a Down (and vice versa); Die Toten Hosen, to Billy Talent and Nirvana; Billy Talent, to Linkin Park, Die Toten Hosen and Muse; Jack Johnson, to Red Hot Chili Peppers and Foo Fighters; Muse, to The Killers; Beatsteaks, to Jack Johnson; and Foo Fighters, to Beatsteaks. Regarding the results based on LR, we mention the following issues. Our influence measure shows that Rammstein is, in this case, the most influential band in listening to Linkin Park and Metallica; System of a Down, to Rammstein; Billy Talent, to Systems of a Down; The Beatles, to Red Hot Chili Peppers; Jack Johnson, to Die Toten Hosen; Foo Fighters, to Coldplay, and finally, Nirvana, to Beatsteaks.

Table 17 shows the Pearson correlations between the influence measures obtained for each band when using RF and LR as classifiers. In this case, the greatest similarities between the rankings are found, in this order, when we examine the influence of listening to Rammstein, Red Hot Chili Peppers, and Coldplay.

Finally, one aspect to be considered is the possible effect of partitioning on the resulting ranking. Although some similarities can apparently be observed between the rankings obtained under CDP and HCP, we determine the corresponding Pearson correlations in Table 18 between the rankings obtained when using RF as classifier as well as the case of

RF	linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
linkin.park	-	0.0000	0.0028	0.5128	<b>1.0256</b>	<b>0.1114</b>	<b>0.8350</b>	0.0282	0.0000	0.0000	0.0028	0.0090	0.0271	0.0000	0.0000
coldplay	0.0000	-	0.0693	0.0241	0.0271	0.0000	0.0226	0.0367	0.0905	0.0000	0.0056	0.0090	0.0211	0.0754	0.0000
red.hot.chili.peppers	0.0000	0.0000	-	0.3110	0.0000	0.0000	0.0078	0.0038	0.0000	0.0117	0.0109	0.0113	0.0000	0.0000	0.0313
rammstein	0.0000	0.0030	0.0166	-	0.7903	0.0167	0.6996	<b>0.0423</b>	0.0082	0.0000	0.0056	0.0000	0.0030	0.0030	0.0057
system.of.a.down	0.0000	0.0121	0.0111	<b>0.7149</b>	-	0.0251	0.1128	0.0000	0.0137	0.0000	0.0084	0.0000	0.0121	0.0060	0.0115
metallica	0.0000	0.0000	0.0156	0.4205	0.0000	-	0.0078	0.0000	0.0000	0.0117	0.0219	0.0226	0.0000	0.0000	0.0234
die.toten.hosen	0.0000	0.0000	0.0078	0.5512	0.0000	0.0000	-	0.0075	0.0000	0.0117	0.0073	0.0038	0.0000	0.0000	<b>0.0469</b>
billy.talent	<b>19.2817</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0988	-	0.0000	0.0000	0.0000	0.0000	<b>0.1107</b>	0.0000	0.0040
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0277	0.0000	-	0.0000	0.0000	0.0277	0.0000	0.0000	0.0000
the.beatles	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0454	0.0000	0.0000	0.0038	0.0000	0.0039	0.0040	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000
muse	0.0090	0.0090	0.0804	0.0000	0.0000	0.0474	0.0310	0.0226	<b>0.0768</b>	0.0000	0.0253	-	<b>0.0483</b>	0.0090	0.0345
beatsteaks	0.0181	0.0151	<b>0.1304</b>	0.0030	0.0000	0.0362	0.0395	0.0339	<b>0.2936</b>	<b>0.2331</b>	0.0000	0.0362	-	<b>0.1388</b>	0.0000
foo.fighters	0.0000	<b>0.0392</b>	0.0832	0.0030	0.0060	0.0084	0.0000	0.0169	0.0165	0.0000	0.0281	0.0090	0.0121	-	0.0029
nirvana	0.0000	0.0000	0.0352	0.2014	0.0000	0.0000	0.0469	0.0113	0.0000	<b>0.0156</b>	0.0036	0.0038	0.0000	0.0000	-

  

SVM	linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
linkin.park	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
coldplay	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
red.hot.chili.peppers	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
rammstein	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
system.of.a.down	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
metallica	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
die.toten.hosen	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
billy.talent	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.beatles	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000
muse	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000
beatsteaks	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000
foo.fighters	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
nirvana	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-

  

LR	linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
linkin.park	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
coldplay	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
red.hot.chili.peppers	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0078	0.0000	0.0000	0.0000	0.0000	0.0000
rammstein	0.0000	0.0000	0.0000	-	0.0000	<b>0.0056</b>	<b>0.0056</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
system.of.a.down	0.0000	0.0000	0.0000	0.0965	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
metallica	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	<b>0.0117</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
die.toten.hosen	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
billy.talent	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.beatles	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000
muse	0.0000	0.0000	0.0000	<b>0.1448</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000
beatsteaks	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0.0027</b>	0.0000	0.0000	0.0000	-	<b>0.0030</b>	0.0000
foo.fighters	0.0000	<b>0.0030</b>	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0.0028</b>	0.0000	0.0000	0.0000	<b>0.0030</b>	<b>0.0030</b>	-	0.0000
nirvana	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0039	0.0000	0.0000	0.0000	0.0000	-

Table 14: Influence measure (3) using the constructed-by-design partition in the analysis of musical taste in Spotify.

linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
-	0.8824	-	0.1451	-	-0.0099	0.5984	0.0448	0.9290	0.7143	-	-0.0822	0.2272	0.8704	-

Table 15: Correlations between the rankings obtained for RF and LR with CDP.

	linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
RF															
linkin.park	-	0.3989	0.0000	1.2427	1.1437	0.0031	0.0269	0.0062	0.0078	0.0000	0.0082	0.0138	0.0126	0.0000	0.0211
coldplay	0.0000	-	0.5779	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
red.hot.chili.peppers	0.0093	<b>0.8825</b>	-	0.3369	0.1669	<b>0.4822</b>	0.0000	0.0309	0.0117	0.0000	0.0408	0.0055	0.0067	0.0038	0.0070
rammstein	0.0556	0.0328	0.0335	-	<b>1.4807</b>	0.1669	0.0000	0.0340	0.0000	0.0000	0.0082	0.0083	0.0000	0.0000	0.0000
system.of.a.down	0.0216	0.0383	0.1673	<b>1.4683</b>	-	0.0433	0.0000	0.1453	0.0000	0.0000	0.0163	0.0083	0.0000	0.0000	0.0000
metallica	0.0247	0.0301	0.0000	1.2674	1.1376	-	0.0000	0.0309	0.0000	0.0000	0.0245	0.0083	0.0034	0.0038	0.0000
die.toten.hosen	0.0309	0.0055	0.1673	1.2798	0.2844	0.0185	-	<b>0.1484</b>	0.0156	0.0000	0.0109	0.0166	0.0000	0.0000	<b>0.0352</b>
billy.talent	<b>21.3682</b>	0.0820	0.0000	0.1329	0.2658	0.0464	<b>0.0960</b>	-	0.0000	0.0000	0.0082	<b>0.0775</b>	0.0502	0.0000	0.0282
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0307	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0045	0.0000
the.beatles	0.0000	0.0277	0.0608	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0000	0.0000	<b>1.3080</b>	0.0037	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	<b>0.0115</b>	0.0000
muse	0.0042	0.0000	0.0078	0.0000	0.0000	0.0000	0.0192	0.0000	<b>0.0165</b>	0.0000	0.0000	-	0.0000	0.0000	0.0000
beatsteaks	0.0126	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0045	0.0000	<b>0.3152</b>	0.0000	<b>0.0775</b>	-	0.0000	0.0070
foo.fighters	0.1004	0.0285	0.0044	0.0000	0.0000	0.0043	0.0000	0.0000	0.0041	0.0000	0.0162	0.0000	<b>0.0753</b>	-	0.0070
nirvana	0.1235	0.0000	0.0000	0.0000	0.0000	0.0000	0.0531	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-
SVM															
linkin.park	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
coldplay	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
red.hot.chili.peppers	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
rammstein	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
system.of.a.down	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
metallica	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
die.toten.hosen	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
billy.talent	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.beatles	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000
muse	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000
beatsteaks	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000
foo.fighters	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
nirvana	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-
LR															
linkin.park	-	0.0000	0.0000	0.0185	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
coldplay	0.0000	-	0.0532	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
red.hot.chili.peppers	0.0000	0.0219	-	0.0278	0.0031	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
rammstein	<b>0.0031</b>	0.0000	0.0000	-	0.0124	<b>0.0093</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
system.of.a.down	0.0000	0.0000	0.0000	<b>0.7913</b>	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
metallica	0.0000	0.0027	0.0000	0.4884	0.0031	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
die.toten.hosen	0.0000	0.0000	0.0000	0.3369	0.0031	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
billy.talent	0.0000	0.0000	0.0000	0.0309	<b>0.0402</b>	0.0062	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.killers	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
the.beatles	0.0000	0.0000	<b>0.0608</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000	0.0000
jack.johnson	0.0000	0.0000	0.0456	0.0000	0.0000	0.0000	<b>0.0036</b>	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000	0.0000
muse	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000	0.0000
beatsteaks	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000	0.0000
foo.fighters	0.0000	<b>0.0285</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	0.0000
nirvana	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0.0067</b>	0.0000	-

Table 16: Influence measure (3) using the hierarchical clustering partition in the analysis of musical taste in Spotify.

linkin.park	coldplay	red.hot.chili.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
-0.0756	0.4911	0.6255	0.8122	0.1923	0.2026	-0.1640	-	-	-	-	-	-0.1328	-	-

Table 17: Correlations between the rankings obtained for RF and LR with HCP.

considering LR.

	linkin.park	coldplay	red.hot.chilli.peppers	rammstein	system.of.a.down	metallica	die.toten.hosen	billy.talent	the.killers	the.beatles	jack.johnson	muse	beatsteaks	foo.fighters	nirvana
RF	0.9999	-0.1964	-0.1546	0.9484	0.7776	-0.1768	0.0469	-0.2691	-0.1001	-	0.9859	0.8619	-0.0137	-0.2259	0.3248
LR	-	0.7746	-	0.3316	-	0.8190	-0.0769	-	-	-	-	-	-0.1132	-	-

Table 18: Correlations between the rankings obtained under CDP and HCP.

When using RF, the highest similarities in the rankings are found for Linkin Park, Jack Johnson, Rammstein, Muse and System of a Down, with correlations above 0.75. In these cases, the use of different partitions on the features causes small numerical variations in the rankings. This is not common for the rest of the bands, where even negative correlations are obtained. Nor is it when using LR as a classifier, where only Metallica and Coldplay exceed the 0.75 correlation threshold.

## 7 Concluding remarks

In this paper, we have introduced a method to analyze the influence of specific features in a classification problem where dependencies might exist between them. Our influence measure extends Datta et al.’s approach to accommodate the existence of a coalitional structure on such attributes. Through properties typical in cooperative game theory, we provide an axiomatic characterization of this measure. Furthermore, we demonstrate that the Banzhaf-Owen value serves as a special case of our measure under conditions of binary databases and no repeated profiles. The methodology presented here is subsequently applied to investigate influential variables in traffic accidents or to analyze musical taste on Spotify.

In this context, several lines of research stemming from our proposal remain open and need to be addressed in the future. From a practical perspective, the methodology presented in this paper can be used in a wide range of real-world applications. One example is the booming sector of streaming platforms such as Amazon Prime, Disney+, Max, or Netflix. It would be particularly interesting to conduct a similar analysis to what we performed for the Spotify database, aiming to identify which audiovisual content has the greatest (or least) impact on the viewing of others. The inherent similarities among some of these media resources make our influence measure with dependencies applicable in this domain. This would enable us to propose personalized recommendations to users, which is of major relevance to platforms’ stakeholders (Bourreau and Gaudin, 2022). However, significant computational challenges arise when applying our methodology to real-world problems due to the sheer and diverse amount of information available in today’s society. Besides, we assumed in Section 3 that each element of a sample  $\mathcal{M}$  appears exactly once. Alternatively, we could consider that each of the  $n$  distinct profiles in the sample,  $(X^i, Y^i)$ ,

with  $i \in N$ , has an absolute frequency of  $n(i)$ , resulting in a total sample size of  $\sum_{i \in N} n(i)$ . By extending the properties used in Theorem 4.3, particularly those of disjoint unions and symmetry, to this context, we can characterize a weighted influence measure that accommodates dependent features and accounts for repeated profiles. Specifically, for each  $l \in K$ , this measure is defined by

$$\hat{I}_l(X, Y, P, \mathcal{M}) = C \cdot \hat{\Psi}_l(X, Y, P, \mathcal{M}) = C \cdot \sum_{\substack{i \in N: \\ (X^i, Y^i) \in \mathcal{M}^t}} \sum_{\substack{((X_{-l}^i, a_l), b) \in \mathcal{M}^t: \\ a_l \in \mathcal{A}_l, b \in \mathcal{B}}} n(i) \cdot |Y^i - b|.$$

Even so, the exact calculation in practice of our proposed influence measure reaches exponential computational complexity as does the Banzhaf-Owen value for a sufficiently large number of players involved (see Deng and Papadimitriou, 1994). To overcome this issue, it seems intuitive that our Banzhaf-Owen value-based influence measure can also be approximated by statistical sampling in large-scale settings, following the ideas in Saavedra-Nieves and Fiestras-Janeiro (2021) for estimating such a game-theoretical solution concept. Finally, we would like to highlight the potential value of conducting a more comprehensive study on the performance of our influence measure using other classifiers in the existing literature. To the best of the authors' knowledge, random forests, support vector machines, and logistic regression-type classifiers are among the most widely used methods for binary data. We have, however, verified the strong dependence of the chosen databases and the response variable determined by a certain classifier. Furthermore, we took base implementations of these methods in the RWeka library. Calibrating different parameters of these estimators, as well as testing other classification methods, might yield more conclusive results for some of the analyzed scenarios.

## Acknowledgments

This work is part of the R+D+I project PID2021-124030NB-C32, granted by MICIU/AEI/10.13039/501100011033/ and by "ERDF A way of making Europe"/EU. This research was also funded by *Grupos de Referencia Competitiva* ED431C 2021/24 from the *Consellería de Cultura, Educación e Universidades, Xunta de Galicia*. Authors also thank the computational resources of the *Centro de Supercomputación de Galicia (CESGA)*.

## Declaration of interest

The authors declare that there is no conflict of interest.

## References

Aas, K., Jullum, M., and Løland, A. (2021). Explaining individual predictions

- when features are dependent: More accurate approximations to Shapley values. *Artificial Intelligence*, 298, 103502.
- Algaba, E., Prieto, A., and Saavedra-Nieves, A. (2024). Risk analysis sampling methods in terrorist networks based on the Banzhaf value. *Risk Analysis*, 44(2), 477–492.
- Alonso-Meijide, J., Carreras, F., Fiestras-Janeiro, G., and Owen, G. (2007). A comparative axiomatic characterization of the Banzhaf-Owen coalitional value. *Decision Support Systems*, 43(3), 701–712.
- Altmann, A., Toloşi, L., Sander, O., and Lengauer, T. (2010). Permutation importance: a corrected feature importance measure. *Bioinformatics*, 26(10), 1340–1347.
- Arévalo-Iglesias, G., and Álvarez-Mozos, M. (2020). Power distribution in the Basque Parliament using games with externalities. *Theory and Decision*, 89(2), 157–178.
- Banzhaf III, J. (1965). Weighted voting does not work: A mathematical analysis. *Rutgers Law Review*, 19(2), 317-343.
- Bourreau, M., and Gaudin, G. (2022). Streaming platform and strategic recommendation bias. *Journal of Economics & Management Strategy*, 31(1), 25–47.
- Breiman, L. (2001). Random forests. *Machine Learning*, 45, 5–32.
- Burkart, N., and Huber, M. F. (2021). A survey on the explainability of supervised machine learning. *Journal of Artificial Intelligence Research*, 70, 245–317.
- Carrizosa, E., Molero-Río, C., and Morales, D. R. (2021). Mathematical optimization in classification and regression trees. *TOP*, 29, 5–33.
- Casalicchio, G., Molnar, C., and Bischl, B. (2019). Visualizing the feature importance for black box models. In *Berlingiero, M., Bonchi, F., Gärtner, T., Hurley, N., and Ifrim, G. (Eds.), Machine Learning and Knowledge Discovery in Databases* (pp. 655–670). Springer.
- Cohen, S., Dror, G., and Ruppin, E. (2007). Feature selection via coalitional game theory. *Neural Computation*, 19(7), 1939–1961.

- Datta, A., Datta, A., Procaccia, A., and Zick, Y. (2015). Influence in classification via cooperative game theory. *Proceedings of the Twenty-fourth International Joint Conference on Artificial Intelligence*, 511–517.
- Davila-Pena, L., García-Jurado, I., and Casas-Méndez, B. (2022). Assessment of the influence of features on a classification problem: An application to COVID-19 patients. *European Journal of Operational Research*, 299(2), 631–641.
- Deng, X., and Papadimitriou, C. H. (1994). On the complexity of cooperative solution concepts. *Mathematics of Operations Research*, 19(2), 257–266.
- European Commission. (2020). *White Paper on Artificial Intelligence: a European approach to excellence and trust*. Retrieved from <https://commission.europa.eu/publications/white-paper-artificial-intelligence-european> (Accessed: July 17, 2024)
- Feltkamp, V. (1995). Alternative axiomatic characterization of the Shapley and Banzhaf values. *International Journal of Game Theory*, 24(2), 179–186.
- Fernández-Delgado, M., Cernadas, E., Barro, S., and Amorim, D. (2014). Do we need hundreds of classifiers to solve real world classification problems? *The Journal of Machine Learning Research*, 15(1), 3133–3181.
- Ghaddar, B., and Naoum-Sawaya, J. (2018). High dimensional data classification and feature selection using support vector machines. *European Journal of Operational Research*, 265(3), 993–1004.
- Giudici, P., and Raffinetti, E. (2021). Shapley-Lorenz explainable artificial intelligence. *Expert Systems with Applications*, 167, 114104.
- Jothi, N., Husain, W., and Rashid, N. (2021). Predicting generalized anxiety disorder among women using Shapley value. *Journal of Infection and Public Health*, 14, 103–108.
- le Cessie, S., and van Houwelingen, J. (1992). Ridge estimators in logistic regression. *Applied Statistics*, 41(1), 191–201.
- Lehrer, E. (1988). An axiomatization of the Banzhaf value. *International Journal of Game Theory*, 17(2), 89–99.

- Li, M., Sun, H., Huang, Y., and Chen, H. (2024). Shapley value: from cooperative game to explainable artificial intelligence. *Autonomous Intelligent Systems*, 4, 2.
- Lundberg, S., and Lee, S. (2017). A unified approach to interpreting model predictions. In *Guyon, I., Luxburg, U. V., Bengio, S., Wallach, H., Fergus, R., Vishwanathan, S. and Garnett, R. (Eds.), Advances in Neural Information Processing Systems* (Vol. 30, pp. 4765–4774). Curran Associates Inc.
- Muros, F., Algaba, E., Maestre, J. M., and Camacho, E. F. (2017). Banzhaf value as a design tool in coalitional control. *Systems & Control Letters*, 104, 21–30.
- Nahiduzzaman, M., Faisal Abdulrazak, L., Arselene Ayari, M., Khandakar, A., and Islam, S. R. (2024). A novel framework for lung cancer classification using lightweight convolutional neural networks and ridge extreme learning machine model with SHapley Additive exPlanations (SHAP). *Expert Systems with Applications*, 248, 123392.
- Olsen, L. H. B., Glad, I. K., Jullum, M., and Aas, K. (2022). Using Shapley values and variational autoencoders to explain predictive models with dependent mixed features. *Journal of Machine Learning Research*, 23(213), 1–51.
- Owen, A. B., and Prieur, C. (2017). On Shapley value for measuring importance of dependent inputs. *SIAM/ASA Journal on Uncertainty Quantification*, 5(1), 986–1002.
- Owen, G. (1977). Values of games with a priori unions. In *Henn, R., Moeschlin, O. (eds.) Mathematical Economics and Game Theory* (pp. 76–88). Springer.
- Owen, G. (1981). Modification of the Banzhaf-Coleman index for games with a priori unions. In *Holler, M. J. (ed.) Power, Voting, and Voting Power* (pp. 232–238). Springer.
- Platt, J. (1998). Fast training of support vector machines using sequential minimal optimization. In *B. Schoelkopf, C. Burges, and A. Smola (Eds.), Advances in Kernel Methods - Support Vector Learning*. MIT Press.
- Saavedra-Nieves, A., and Fiestras-Janeiro, M. G. (2021). Sampling methods to estimate the Banzhaf–Owen value. *Annals of Operations Research*, 301(1), 199–223.
- Shapley, L. (1953). A value for n-person games. *Annals of Mathematics Studies*, 28(7), 307–317.

- Smith, M., and Alvarez, F. (2021). Identifying mortality factors from Machine Learning using Shapley values – a case of COVID19. *Expert Systems with Applications*, 176, 114832.
- Štrumbelj, E., and Kononenko, I. (2010). An efficient explanation of individual classifications using game theory. *Journal of Machine Learning Research*, 11(1), 1–18.
- Štrumbelj, E., and Kononenko, I. (2011). A general method for visualizing and explaining black-box regression models. In *Dobnikar, A., Lotrič, U., Šter, B. (Eds.), Adaptive and Natural Computing Algorithms* (pp. 21–30). Springer.